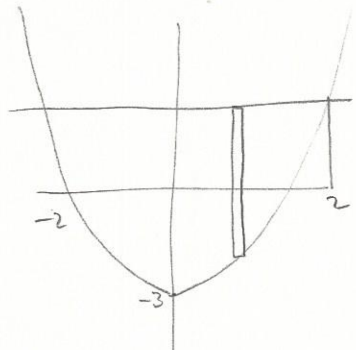
1. Find the area enclosed by the parabola $y = x^2 - 3$ and the line $y = 1$.

a) $\frac{8}{3}$ b) 32 c) $\frac{32}{3}$ d) $\frac{16}{3}$ e) none of these



$1 = x^2 - 3$

$4 = x^2$

$\pm 2 = x$

$\int_{-2}^{2} (1-(x^2-3))dx$

$2\int_{0}^{2} (1-x^2+3)\,dx$

$2\int_{0}^{2} (4-x^2)dx$

$2(4x - \frac{1}{3}x^3]_0^2$

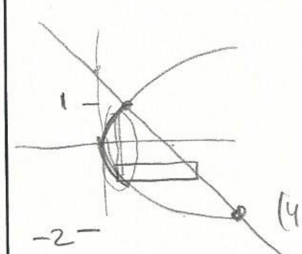
$2(8-\frac{8}{3}) \to 2(\frac{24}{3}-\frac{8}{3}) \Rightarrow 2\,\frac{16}{3} = \frac{32}{3}$

$y = \pm\sqrt{x}$

2. Find the area enclosed by the parabola $y^2 = x$ and the line $x + y = 2$. $\Rightarrow x = 2 - y$

$y = -x + 2$

a) $\frac{5}{2}$ b) $\frac{3}{2}$ c) $\frac{11}{6}$ d) $\frac{9}{2}$ e) $\frac{29}{6}$



$y^2 = 2 - y$

$y^2 + y - 2 = 0$

$(y+2)(y-1) =$

$\int_{-2}^{1} ((2-y) - y^2)dy$

$= 2y - \frac{1}{2}y^2 - \frac{1}{3}y^3]_{-2}^{1}$

$= (2 - \frac{1}{2} - \frac{1}{3}) + (+4 + 2 + -\frac{8}{3})$

$= 8 - \frac{1}{2} - 3$

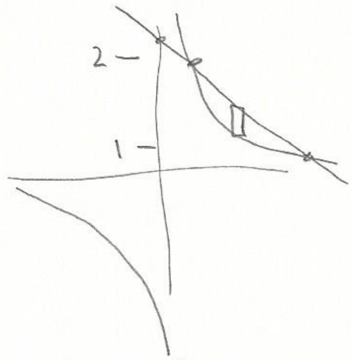
$= 5 - \frac{1}{2} = \frac{9}{2}$

$\int_0^1 2\sqrt{x}\,dx + \int_1^4 ((2-x) - -\sqrt{x})dx$

3. Find the area enclosed by the curve of $y=\frac{2}{x}$ and $x+y=3$.

$y=-x+3$

a) $\frac{1}{2}-2\ln(2)$ b) $\frac{3}{2}$ c) $\frac{1}{2}-\ln(4)$ d) $\frac{5}{2}$ e) $\frac{3}{2}-\ln(4)$



$$\int_1^2\left((3-x)-\frac{2}{x}\right)dx$$

$$3x-\frac{1}{2}x^2-2\ln|x|\Big]_1^2$$

$$(6-2-2\ln 2)+(-3+\tfrac{1}{2}-0)$$

$$1+\tfrac{1}{2}-2\ln 2$$

$$\boxed{\tfrac{3}{2}-\ln 4}$$

$-x+3=\frac{2}{x}$

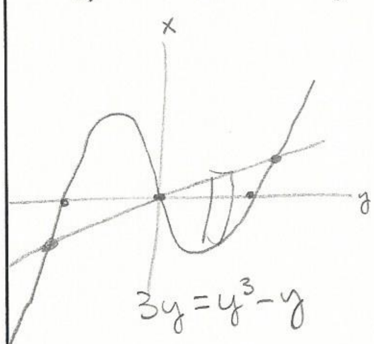
$-x^2+3x=2$

$0=x^2-3x+2 \Rightarrow 0=(x-2)(x-1)$

4. Find the total area bounded by the cubic $x=y^3-y$ and the line $x=3y$:

$y(y^2-1)$

a) 4 b) $\frac{16}{3}$ c) 8 d) $\frac{32}{3}$ e) 16



$$2\int_0^2\left(3y+(-y^3+y)\right)dy$$

$$2\int_0^2(4y-y^3)\,dy$$

$$2\left(2y^2-\tfrac{1}{4}y^4\right]_0^2$$

$$2((8-4)-(0-0))$$

$$8$$

$3y=y^3-y$

$0=y^3-4y$

$0=y(y^2-4)$

$y=0,\ y=\pm 2$

5. Suppose the following is a table of values for $y = f(x)$, given that f is continuous on [1,5]:

x	1	2	3	4	5
y	1.62	4.15	7.50	9.00	12.13

If a trapezoidal sum is used, with $n = 4$, then the area under the curve from $x = 1$ to $x = 5$ is equal, to two decimal places, to.....

a) 6.88 b) 13.76 c) 20.30 d) 25.73 e) 27.53

$$A = \frac{1}{2}(1.62+4.15)1 + \frac{1}{2}(4.15+7.5)1 + \frac{1}{2}(7.5+9)1 + \frac{1}{2}(9+12.13)1$$

$$= \frac{1}{2}\cdot 1.62 + 4.15 + 7.5 + 9 + \frac{1}{2}\cdot 12.13$$

$$= 27.525$$

6. Find the volume of the solid formed when the first quadrant region bounded by $y = x^2$, the y-axis, and $y = 4$, are revolved about the y-axis.

a) 8π b) 4π c) $\frac{64\pi}{3}$ d) $\frac{32\pi}{3}$ e) $\frac{16\pi}{3}$

$$\int_0^4 \pi (\sqrt{y})^2 \, dy$$

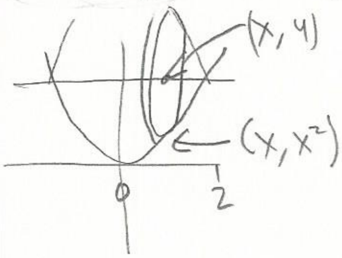
$$\pi \int_0^4 y \, dy$$

$$\frac{\pi}{2} y^2 \Big]_0^4$$

$$8\pi$$

7. Find the volume of the solid formed when the region enclosed by the curves $y = x^2$ and $y = 4$ is revolved about the line $y = 4$.

a) $\frac{256\pi}{15}$ b) $\frac{256\pi}{5}$ c) $\frac{512\pi}{5}$ d) $\frac{512\pi}{15}$ e) $\frac{64\pi}{3}$



$$2\int_0^2 \pi\left((4-x^2)\right)^2 dx$$

$$2\pi\int_0^2 (16-8x^2+x^4)\,dx$$

$$2\pi\left(16x-\frac{8}{3}x^3+\frac{1}{5}x^5\right]_0^2$$

$$2\pi\left(\left(32-\frac{64}{3}+\frac{32}{5}\right)-(0-0+0)\right)$$

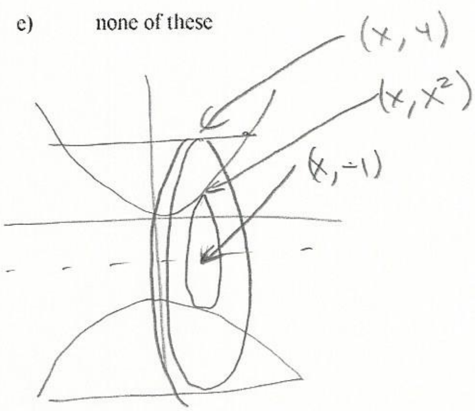
$$2\pi\left(\frac{480}{15}-\frac{320}{15}+\frac{96}{15}\right)=\frac{512}{15}\pi$$

$$\begin{array}{r} 160 \\ 96 \\ \hline 256 \end{array} \qquad \begin{array}{r} 32 \\ 15 \\ \hline 32 \\ 160 \\ 480 \end{array}$$

8. The integral set-up for the volume formed when the region enclosed by the curves $y = x^2$ and $y = 4$ is revolved about the line $y = -1$ would be:

a) $4\pi\int_{-1}^{4}(y+1)\sqrt{y}\,dy$ b) $2\pi\int_0^2(4-x^2)^2dx$ c) $\pi\int_{-2}^{2}(16-x^4)dx$ d) $2\pi\int_0^2(24-2x^2-x^4)dx$

e) none of these



$$2\int_0^2 \pi\,(R^2-r^2)\,dx$$

$$2\pi\int_0^2\left((5)^2-(x^2--1)^2\right)dx$$

$$2\pi\int_0^2\left(25-(x^2+1)^2\right)dx$$

$$2\pi\int_0^2\left(25-(x^4+2x^2+1)\right)dx$$

$$2\pi\int_0^2(24-x^4-2x^2)\,dx$$

9. The integral set-up for the volume enclosed by the curves $y = 3x - x^2$ and $y = x$ about the x-axis would be:

$y = x(3-x)$

a) $\pi\int_0^{3/2}\left[(3x-x^2)^2 - x^2\right]dx$

b) $\pi\int_0^{2}\left(9x^2 - 6x^3\right)dx$

c) $\pi\int_0^{2}\left[(3x-x^2)^2 - x^2\right]dx$

d) $\pi\int_0^{3}\left[(3x-x^2)^2 - x^4\right]dx$

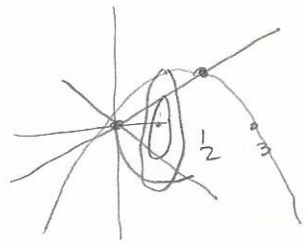
e) $\pi\int_0^{3}\left(2x - x^2\right)^2 dx$

$x = 3x - x^2$

$x^2 - 3x + x = 0$

$x - 2x = 0$

$x(x-2)$



$\int_0^2 \pi(R^2 - r^2)\,dx$

$\int_0^2 \pi\left((3x-x^2)^2 - (x)^2\right)dx$

10. The integral set-up for the volume enclosed by the curves $y = \ln(x)$, $y = 0$, and $x = e$ about the line $x = e$ would be:

$e^y = x$

a) $\pi\int_1^{e}(e - x)\ln(x)\,dx$

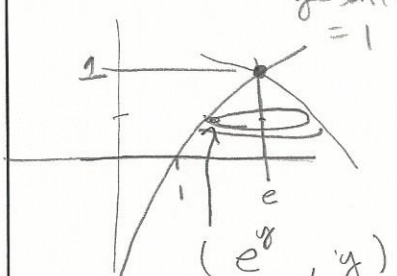
b) $\pi\int_0^{1}\left(e - e^y\right)^2 dy$

c) $2\pi\int_1^{e}(e - \ln(x))\,dx$

d) $\pi\int_0^{e}\left(e^2 - 2e^{y+1} + e^{2y}\right)dy$

e) none of these

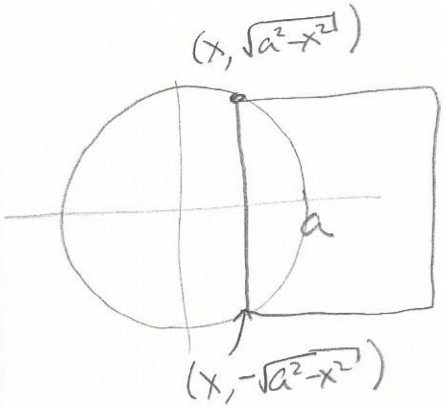
$y = \ln(e)$
$= 1$



$\int_0^1 \pi\left(e - e^y\right)^2 dy$

11. The base of a solid is a circle of radius a, and every plane section perpendicular to a diameter is a square. The solid has volume

a) $\frac{8}{3}a^3$ b) $2\pi a^3$ c) $4\pi a^3$ d) $\frac{16}{3}a^3$ e) $\frac{8\pi}{3}a^3$



$$= \int_{-a}^{a} \left(2\sqrt{a^2-x^2}\right)^2 dx$$

$$= 2\int_0^a 4(a^2-x^2)\,dx$$

$$= 8\left(a^2x - \tfrac{1}{3}x^3\right]_0^a$$

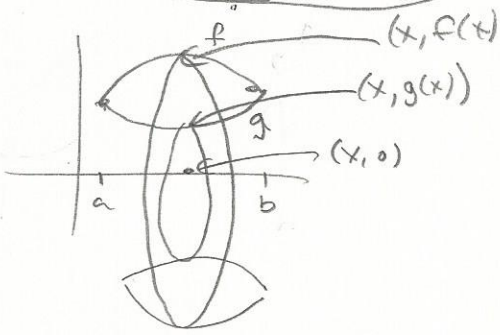
$$= 8\left(\left(a^3 - \tfrac{1}{3}a^3\right) - (0-0)\right)$$

$$= 8\cdot\tfrac{2}{3}a^3 = \tfrac{16}{3}a^3$$

12. If the curves of $f(x)$ and $g(x)$ intersect for $x = a$ and $x = b$ and if $f(x) > g(x) > 0$ for all x on (a,b), then the volume obtained when the region bounded by the curves is rotated about the x-axis is equal to

a) $\pi\int_a^b f^2(x)dx - \int_a^b g^2(x)dx$ b) $\pi\int_a^b [f(x)-g(x)]^2\,dx$ c) $2\pi\int_a^b x[f(x)-g(x)]dx$

d) $\pi\int_a^b [f^2(x)-g^2(x)]dx$ e) none of these

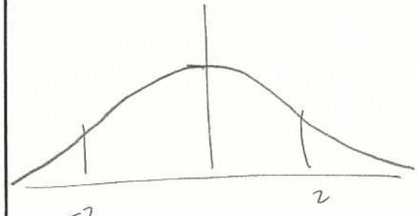


$$\int_a^b \pi(R^2-r^2)\,dx$$

$$\pi\int_a^b (f^2-g^2)\,dx$$

13. Find the area enclosed by the curve of $y = \dfrac{4}{x^2+4}$, the x-axis, and the vertical lines $x = -2$ and $x = 2$.

a) $\dfrac{\pi}{4}$ b) $\dfrac{\pi}{2}$ c) 2π d) π e) none of these



$$\int_{-2}^{2} \frac{4}{x^2+4}\,dx = 4\int_{-2}^{2} \frac{1}{x^2+4}\,dx$$

$$= 8\int_{0}^{2} \frac{1}{x^2+4}\,dx = 8 \cdot \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big]_0^2$$

$$= 4 \cdot \tan^{-1}(1) - 4\tan^{-1}(0)$$

$$= 4 \cdot \frac{\pi}{4} = \pi$$

$$4 \cdot \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big]_{-2}^{2}$$

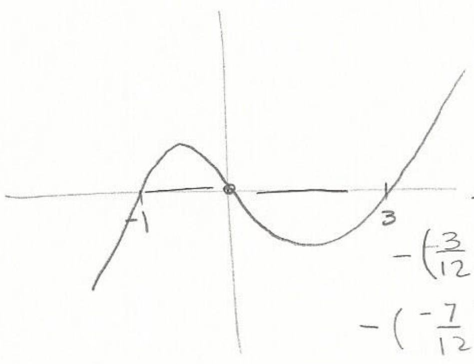
$$2\left(\tan^{-1}(1) - \tan^{-1}(-1)\right)$$

$$2\left(\frac{\pi}{4} + + \frac{\pi}{4}\right) = 2 \cdot \frac{2\pi}{4} = \pi$$

14. Find the area enclosed by the curve $y = x^3 - 2x^2 - 3x$ and the x-axis.

$$y = x(x^2-2x-3) = x(x-3)(x+1)$$

a) $\dfrac{28}{3}$ b) $\dfrac{79}{6}$ c) $\dfrac{45}{4}$ d) $\dfrac{71}{6}$ e) none of these



$$\int_{-1}^{0}(x^3-2x^2-3x)\,dx + \int_{0}^{3}-(x^3-2x^2-3x)\,dx$$

$$\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2\Big]_{-1}^{0} + \int_0^3(-x^3+2x^2+3x)\,dx$$

$$-\left(\frac{1}{4}+\frac{2}{3}-\frac{3}{2}\right) + -\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{3}{2}x^2\Big]_0^3$$

$$-\left(\frac{3}{12}+\frac{8}{12}-\frac{18}{12}\right) + -\frac{81}{4} + \frac{54}{3} + \frac{27}{2}$$

$$-\left(\frac{-7}{12}\right) + \frac{-243}{12} + \frac{216}{12} + \frac{162}{12}$$

$$\frac{7}{12} + \frac{135}{12} \longrightarrow \frac{142}{12} \to \frac{71}{6}$$

15. The area bounded by the parabola $y = 2 - x^2$ and the line $y = x - 4$ is given by

a) $\int_{-2}^{3} (6 - x - x^2)\,dx$

b) $\int_{-2}^{1} (2 + x + x^2)\,dx$

c) $\int_{-3}^{2} (6 - x - x^2)\,dx$

d) $2\int_{0}^{\sqrt{2}} (2 - x^2)\,dx + \int_{-3}^{2} (4 - x)\,dx$

e) none of these

$2 - x^2 = x - 4$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

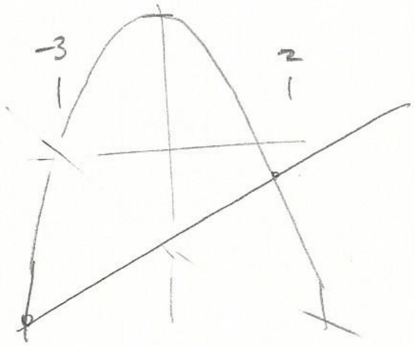
$-3 \quad +2$

$2 - x^2 = x - 4$

$0 = x^2 + x - 6$

$= (x-2)(x+3)$

$x = 2,\ x = -3$

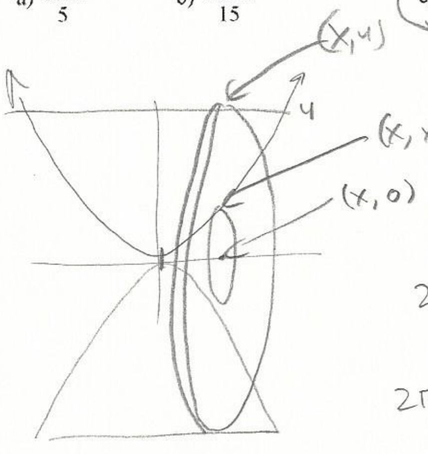


$\int_{-3}^{2} (2-x^2) - (x-4)\,dx$

$\int_{-3}^{2} (2 - x^2 - x + 4)\,dx$

16. The volume of the solid formed when the region bounded by the curve $y = x^2$ and the line $y = 4$ is revolved around the x-axis would be:

a) $\frac{64\pi}{5}$ b) $\frac{512\pi}{15}$ c) $\frac{256\pi}{5}$ d) $\frac{128\pi}{5}$ e) none of these



$\pi \int_{-2}^{2} (R^2 - r^2)\,dx$

$2\pi \int_{0}^{2} (16 - (x^2)^2\,dx$

$2\pi \left(16x - \frac{1}{5}x^5\right]_0^2$

$2\pi \left(\left(32 - \frac{32}{5}\right) - (0-0)\right)$

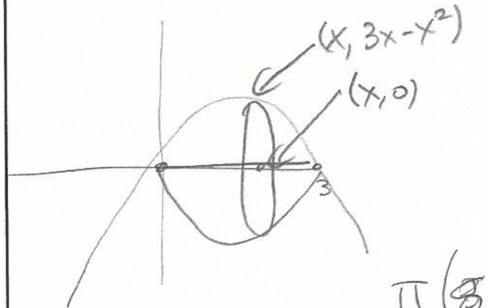
$2\pi \left(\frac{160}{5} - \frac{32}{5}\right) = 2\pi\left(\frac{128}{5}\right) = \frac{256\pi}{5}$

17. The volume of the solid formed when the region bounded by the curve $y = 3x - x^2$ and the line $y = 0$ is revolved around the x-axis would be:

$y = x(3-x)$

a) $\pi\int_0^3 (9x^2 + x^4)dx$ b) $\pi\int_0^3 (3x - x^2)^2 dx$ c) $\pi\int_0^{\sqrt{3}} (3x - x^2)dx$

d) $2\pi\int_0^3 y\sqrt{9-4y}dy$ e) $\pi\int_0^{9/4} y^2 dy$



$\pi\int_0^3 (3x-x^2)^2 dx$

$\pi\int_0^3 (9x^2-6x^3+x^4)dx$

$\pi\left(\frac{9}{3}x^3-\frac{6}{4}x^4+\frac{1}{5}x^5\right)\Big]_0^3$

$\pi\left(81+\frac{3}{2}81-\frac{81}{5}\right)$

$\pi\left(\frac{810}{10}+\frac{1215}{10}-\frac{162}{10}\right)$

$243 \times 5 = 1215$

$2025 - 162$

18. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is

$x = \pm\sqrt{8y}$

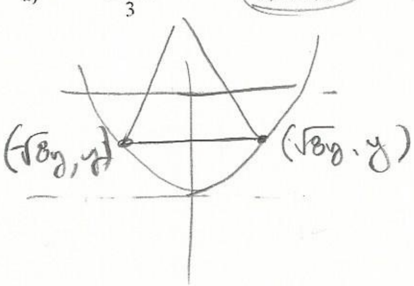
a) $\frac{64\sqrt{3}}{3}$ b) $64\sqrt{3}$ c) $32\sqrt{3}$ d) 32 e) none of these

$A_\Delta = \frac{1}{2}b\cdot h$

$= \frac{1}{2}b\cdot\frac{\sqrt{3}}{2}b$

$= \frac{\sqrt{3}}{4}b^2$

$x^2 = 8\cdot 4$



$\int_0^4 \frac{\sqrt{3}}{4}(2\sqrt{8y})^2 dy$

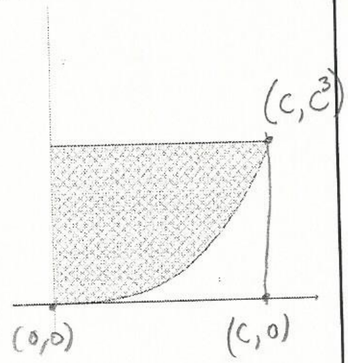
$\sqrt{3}\int_0^4 8y\,dy$

$8\sqrt{3}\int_0^4 y\,dy \rightarrow \frac{8\sqrt{3}}{4}\cdot\frac{1}{2}y^2\Big]_0^4$

$4\sqrt{3}\cdot 16 = 64\sqrt{3}$

19. The figure below shows part of the curve of and a rectangle with two vertices at (0,0) and (c,0). What is the ratio of the area of the rectangle to the shaded part of it above the cubic?

a) 3:4 b) 5:4 c) 4:3 d) 3:1 e) 2:1



Rect $= c \cdot c^3 = c^4$

$$\int_0^c (c^3 - x^3)\,dx$$

$$= xc^3 - \frac{1}{4}x^4\Big]_0^c$$

$$= c^4 - \frac{1}{4}c^4$$

$$= \frac{3}{4}c^4$$

$$\frac{c^4}{\frac{3}{4}c^4} = 4/3$$