1. $\int_1^2 \frac{dz}{3-z} =$

(A) $-\ln 2$ (B) $\frac{3}{4}$ (C) $2\left(\sqrt{2}-1\right)$ (D) $\frac{1}{2}\ln 2$ (E) $\ln 2$

2. The integral $\int_{-4}^{4} \sqrt{16-x^2}\,dx$ gives the area of

(A) a circle of radius 4 (B) a semicircle of radius 4

(C) a quadrant of a circle of radius 4 (D) an ellipse whose semimajor axis is 4

(E) none of these

3. $\int_0^{\pi} \cos^2\theta \sin\vartheta\, d\vartheta =$

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{2}{3}$ (E) 0

4. $\int_0^{\pi/6} \frac{\cos\theta}{1+2\sin\vartheta} d\vartheta =$

(A) $\ln 2$ (B) $\frac{3}{8}$ (C) $-\frac{1}{2}\ln 2$ (D) $\frac{3}{2}$ (E) $\ln\sqrt{2}$

5. If $f(x)$ is continuous on the interval $a \le x \le b$, if this interval is partitioned into n equal subintervals of length Δx, and if x_k is a number in the kth subinterval, then

$\lim_{n\to\infty} \sum_1^n f(x_k)\Delta x$ is equal to

(A) $f(b)-f(a)$ (B) $F(x)+C$, where $\frac{dF(x)}{dx} = f(x)$ and C is an arbitrary constant

(C) $\int_a^b f(x)\,dx$ (D) $F(b-a)$, where $\frac{dF(x)}{dx} = f(x)$

(E) none of these

6. If $f(x)$ is continuous on the interval $a \le x \le b$ and $a < c < b$, then $\int_c^b f(x)\,dx$ is equal to

(A) $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ (B) $\int_a^c f(x)\,dx - \int_a^b f(x)\,dx$

(C) $\int_a^c f(x)\,dx + \int_b^a f(x)\,dx$ (D) $\int_a^b f(x)\,dx - \int_a^c f(x)\,dx$

(E) $\int_a^c f(x)\,dx - \int_b^c f(x)\,dx$

7. **If $f(x)$ is continuous on $a \le x \le b$, then**

(A) $\int_a^b f(x)\,dx = f(b) - f(a)$

(B) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(C) $\int_a^b f(x)\,dx \ge 0$

(D) $\frac{d}{dx}\int_a^x f(t)\,dt = f'(x)$

(E) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x) - f(a)$

8. If $f(x)$ is continuous on the closed interval $[a,b]$, then there exists at least one number c, $a < c < b$, such that $\int_a^b f(x)\,dx$ is equal to

(A) $\frac{f(c)}{b-a}$

(B) $f'(c)(b-a)$

(C) $f(c)(b-a)$

(D) $\frac{f'(c)}{b-a}$

(E) $f(c)[f(b) - f(a)]$

9. If $f(x)$ is continuous on the closed interval $[a,b]$, and k is a constant, then $\int_a^b kf(x)\,dx$ is equal to

(A) $k(b-a)$

(B) $k[f(b) - f(a)]$

(C) $kF(b-a), where \frac{dF(x)}{dx} = f(x)$

(D) $k\int_a^b f(x)\,dx$

(E) $\left.\frac{[kf(x)]^2}{2}\right]_a^b$

10. $\frac{d}{dt}\int_0^t \sqrt{x^3+1}\,dx =$

(A) $\sqrt{t^3+1}$

(B) $\frac{\sqrt{t^3+1}}{3t^2}$

(C) $\frac{2}{3}(t^3+1)(\sqrt{t^3+1}-1)$

(D) $3x^2\sqrt{x^3+1}$

(E) none of these

11. $\int_0^{\pi/4} \cos^2 \vartheta\; d\vartheta =$

(A) $\frac{1}{2}$

(B) $\frac{\pi}{8}$

(C) $\frac{\pi}{8} + \frac{1}{4}$

(D) $\frac{\pi}{8} + \frac{1}{2}$

(E) $\frac{\pi}{8} - \frac{1}{4}$

12. If $F(u)=\int_1^u \left(2-x^2\right)^3 dx$, then $F'(u)$ is equal to

(A) $-6u(2-u^2)^2$ (B) $\frac{(2-u^2)^4}{4}-\frac{1}{4}$ (C) $(2-u^2)^3-1$

(D) $(2-u^2)^3$ (E) $-2u(2-u^2)^3$

13. $\frac{d}{dx}\int_{\pi/2}^{x^2}\sqrt{\sin t}\,dt$

(A) $\sqrt{\sin t^2}$ (B) $2x\sqrt{\sin x^2}-1$ (C) $\frac{2}{3}(\sin^{3/2} x^2-1)$

(D) $\sqrt{\sin x^2}-1$ (E) $2x\sqrt{\sin x^2}$

14. If we let $x=\tan\vartheta$, then $\int_1^{\sqrt{3}}\sqrt{1+x^2}\,dx$ is equivalent to

(A) $\int_{\pi/4}^{\pi/3}\sec\vartheta\,d\vartheta$ (B) $\int_1^{\sqrt{3}}\sec^3\vartheta\,d\vartheta$ (C) $\int_{\pi/4}^{\pi/3}\sec^3\vartheta\,d\vartheta$

(D) $\int_{\pi/4}^{\pi/3}\sec^2\vartheta\tan\vartheta\,d\vartheta$ (E) $\int_1^{\sqrt{3}}\sec\vartheta\,d\vartheta$

15. If the substitution $u=\sqrt{x+1}$ is used, then $\int_0^3\frac{dx}{x\sqrt{x+1}}$ is equivalent to

(A) $\int_1^2\frac{du}{u^2-1}$ (B) $\int_1^2\frac{2du}{u^2-1}$ (C) $2\int_0^3\frac{du}{(u-1)(u+1)}$

(D) $2\int_1^2\frac{du}{u(u^2-1)}$ (E) $2\int_0^3\frac{du}{u(u-1)}$

16. If $x=4\cos\vartheta$ and $y=3\sin\vartheta$, then $\int_2^4 xy\,dx$ is equivalent to

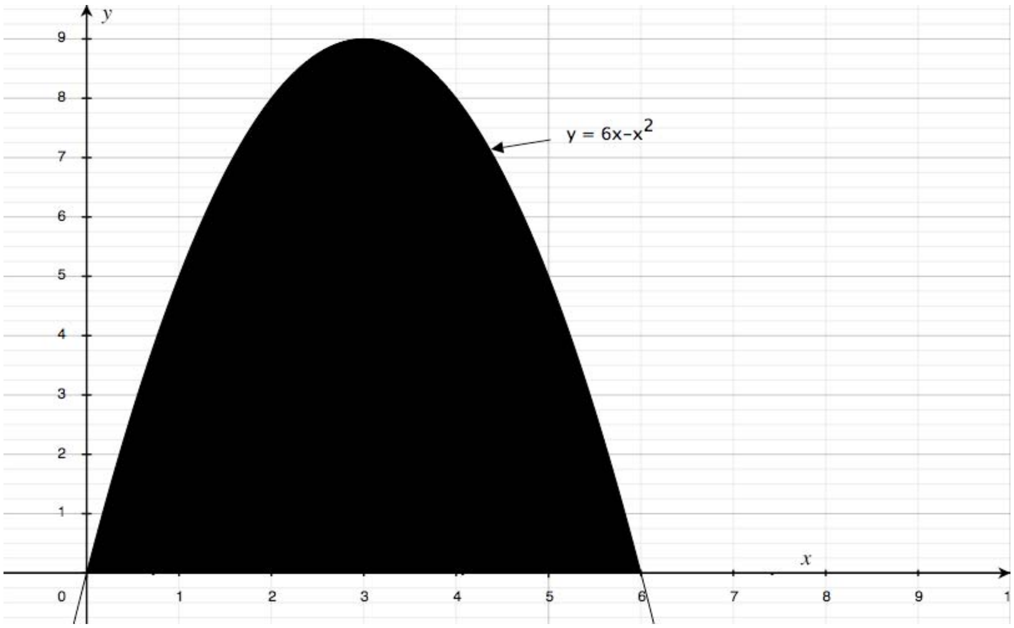
(A) $48\int_{\pi/3}^{0}\sin\vartheta\cos^2\vartheta\,d\vartheta$ (B) $48\int_2^4\sin^2\vartheta\cos\vartheta\,d\vartheta$ (C) $36\int_2^4\sin\vartheta\cos^2\vartheta\,d\vartheta$

(D) $-48\int_0^{\pi/3}\sin\vartheta\cos^2\vartheta\,d\vartheta$ (E) $48\int_0^{\pi/3}\sin^2\vartheta\cos\vartheta\,d\vartheta$

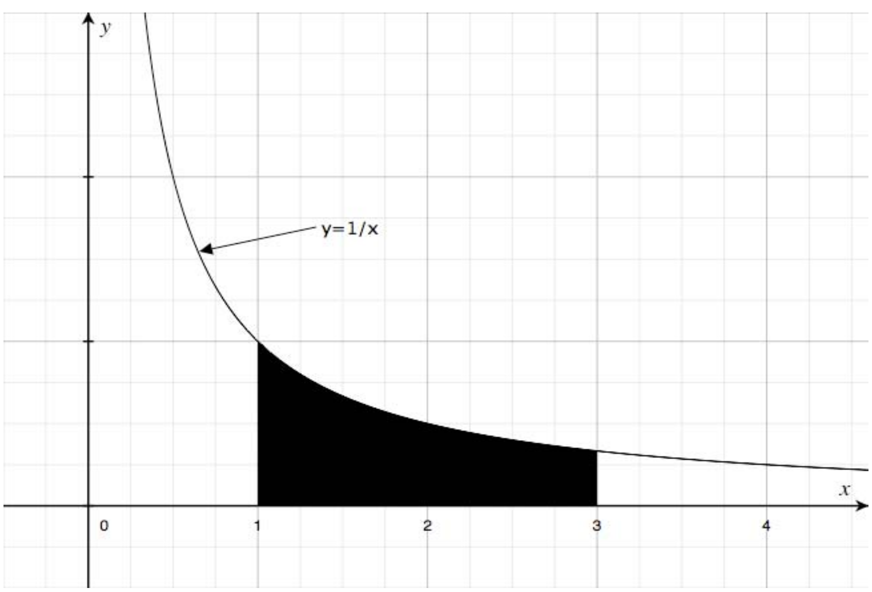
17. If we approximate the area of the shaded region by $M(20)$ (that is, the midpoint sum with 20 subintervals), then the difference $M(20)-\int_0^6 f(x)dx$ is equal to

(A) 0.004 (B) 0.008 (C) 0.010

(D) 0.045 (E) none of these



18. The area of the following shaded region is equal exactly to $\ln 3$. If we approximate $\ln 3$ using $L(2)$ and $R(2)$, which of the inequalities follows?



(A) $\frac{1}{2}<\int_1^2 \frac{1}{x}\,dx<1$ (B) $\frac{1}{3}<\int_1^3 \frac{1}{x}\,dx<2$ (C) $\frac{1}{2}<\int_0^2 \frac{1}{x}\,dx<2$

(D) $\frac{1}{3}<\int_2^3 \frac{1}{x}\,dx<\frac{1}{2}$ (E) $\frac{5}{6}<\int_1^3 \frac{1}{x}\,dx<\frac{3}{2}$

19. If the Trapezoidal Rule is used with $n=5$, then $\int_0^1 \frac{dx}{1+x^2}$ is equal, to three decimal places, to

(A) 0.784 (B) 1.567 (C) 1.959 (D) 3.142 (E) 7.837

20. $\int_{-1}^{3} |x|\, dx =$

(A) $\frac{7}{2}$ (B) 4 (C) $\frac{9}{2}$ (D) 5 (E) $\frac{11}{2}$

21. $\int_{-3}^{2} |x+1|\, dx =$

(A) $\frac{5}{2}$ (B) $\frac{7}{2}$ (C) 5 (D) $\frac{11}{2}$ (E) $\frac{13}{2}$

22. If $M(4)$ is used to approximate $\int_0^1 \sqrt{1+x^3}\, dx$, then the definite integral is equal, to two decimal places, to

(A) 1.00 (B) 1.11 (C) 1.20 (D) 2.22 (E) 3.33

23. $\int_2^5 \frac{1}{3x}\, dx$ is best approximated, to three decimal places, by

(A) 0.268 (B) 0.286 (C) 0.305 (D) 0.916 (E) 2.749

24. The average value of $\cos x$ over the interval $\frac{\pi}{3} \le x \le \frac{\pi}{2}$ is

(A) $\frac{3}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{3(2-\sqrt{3})}{\pi}$ (D) $\frac{3}{2\pi}$ (E) $\frac{2}{3\pi}$

25. The average value of $\csc^2 x$ over the interval from $x=\frac{\pi}{6}$ to $x=\frac{\pi}{4}$ is

(A) $\frac{3\sqrt{3}}{\pi}$ (B) $\frac{\sqrt{3}}{\pi}$ (C) $\frac{12}{\pi}\left(\sqrt{3}-1\right)$ (D) $3\sqrt{3}$ (E) $3(\sqrt{3}-1)$

26. If we let $x = 2\sin\vartheta$ then $\int_1^2 \frac{\sqrt{4-x^2}}{x}\, \mathrm{dx}$ is equivalent to

(A) $2\int_0^2 \frac{\cos^2\vartheta}{\sin\vartheta}\, \mathrm{d}\vartheta$ (B) $\int_{\pi/6}^{\pi/2} \frac{\cos\vartheta}{\sin\vartheta}\, \mathrm{d}\vartheta$ (C) $\int_{\pi/6}^{\pi/2} \frac{\cos^2\vartheta}{\sin\vartheta}\, \mathrm{d}\vartheta$

(D) $\int_2^1 \frac{\cos\vartheta}{\sin\vartheta}\, \mathrm{d}\vartheta$ (E) none of these