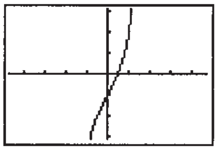
Section 3.8 Derivatives of Inverse Trigonometric Functions (pp. 165–171)

Exploration 1 Finding a derivative on an Inverse Graph Geometrically

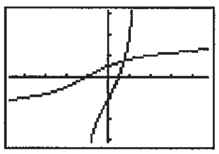
1. The graph is shown at the right. It appears to be a one-to-one function



[−4.7, 4.7] by [−3.1, 3.1]

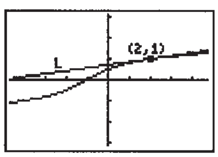
2. $f'(x) = 5x^4 + 2$. The fact that this function is always positive enables us to conclude that f is everywhere increasing, and hence one-to-one.

3. The graph of f^{-1} is shown to the right, along with the graph of f. The graph of f^{-1} is obtained from the graph of f by reflecting it in the line $y = x$.



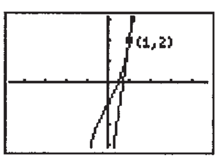
[−4.7, 4.7] by [−3.1, 3.1]

4. The line L is tangent to the graph of f^{-1} at the point $(2, 1)$.



[−4.7, 4.7] by [−3.1, 3.1]

5. The reflection of line L is tangent to the graph of f at the point



[−4.7, 4.7] by [−3.1, 3.1]

6. The reflection of the line L is the tangent line to the graph of $y = x^5 + 2x - 1$ at the point $(1, 2)$. The slope is $\frac{dy}{dx}$ at $x = 1$, which is 7.

7. The slope of L is the reciprocal of the slope of its reflection $\left(\text{since } \frac{\Delta y}{\Delta x} \text{ gets reflected to become } \frac{\Delta x}{\Delta y}\right)$. It is $\frac{1}{7}$.

8. $\frac{1}{7}$

Quick Review 3.8

1. Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

At 1: $\frac{\pi}{2}$

2. Domain: $[-1, 1]$

Range: $[0, \pi]$

At $1: 0$

3. Domain: all reals

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

At $1: \frac{\pi}{4}$

4. Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

At $1: 0$

5. Domain: all reals
Range: all reals
At 1:1

6. $f(x) = y = 3x - 8$

$$y + 8 = 3x$$
$$x = \frac{y+8}{3}$$

Interchange x and y:

$$y = \frac{x+8}{3}$$
$$f^{-1}(x) = \frac{x+8}{3}$$

7. $f(x) = y = \sqrt[3]{x+5}$

$$y^3 = x + 5$$
$$x = y^3 - 5$$

Interchange x and y:

$$y = x^3 - 5$$
$$f^{-1}(x) = x^3 - 5$$

8. $f(x) = y = \frac{8}{x}$

$$x = \frac{8}{y}$$

Interchange x and y:

$$y = \frac{8}{x}$$
$$f^{-1}(x) = \frac{8}{x}$$

9. $f(x) = y = \dfrac{3x-2}{x}$

$$xy = 3x - 2$$
$$(y-3)x = -2$$
$$x = \frac{-2}{y-3} = \frac{2}{3-y}$$

Interchange x and y:

$$y = \frac{2}{3-x}$$
$$f^{-1}(x) = \frac{2}{3-x}$$

10. $f(x) = y = \arctan \dfrac{x}{3}$

$$\tan y = \frac{x}{3},\ -\frac{\pi}{2} < y < \frac{\pi}{2}$$
$$x = 3\tan y,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Interchange x and y:

$$y = 3\tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$f^{-1}(x) = 3\tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Section 3.8 Exercises

1. $$\frac{dy}{dx} = \frac{d}{dx}\cos^{-1}(x^2) = -\frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}(x^2)$$
$$= -\frac{1}{\sqrt{1-x^4}}(2x) = -\frac{2x}{\sqrt{1-x^4}}$$

2. $$\frac{dy}{dx} = \frac{d}{dx}\cos^{-1}\left(\frac{1}{x}\right) = -\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\frac{d}{dx}\left(\frac{1}{x}\right)$$
$$= -\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(-\frac{1}{x^2}\right) = \frac{1}{|x|\sqrt{x^2-1}}$$

3. $$\frac{dy}{dt} = \frac{d}{dt}\sin^{-1}\sqrt{2}t = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}}\frac{d}{dt}(\sqrt{2}t) = \frac{\sqrt{2}}{\sqrt{1-2t^2}}$$

4. $$\frac{dy}{dt} = \frac{d}{dt}\sin^{-1}(1-t) = \frac{1}{\sqrt{1-(1-t)^2}}\frac{d}{dt}(1-t)$$
$$= -\frac{1}{\sqrt{2t-t^2}}$$

5. $$\frac{dy}{dt} = \frac{d}{dt}\sin^{-1}\left(\frac{3}{t^2}\right) = \frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\frac{d}{dt}\left(\frac{3}{t^2}\right)$$
$$= \frac{1}{\sqrt{1-\frac{9}{t^4}}}\left(-\frac{6}{t^3}\right) = -\frac{6}{t\sqrt{t^4-9}}$$

6. $$\frac{dy}{ds} = \frac{d}{ds}(s\sqrt{1-s^2}) + \frac{d}{ds}(\cos^{-1} s)$$
$$= (s)\left(\frac{1}{2\sqrt{1-s^2}}\right)(-2s) + (\sqrt{1-s^2})(1) - \frac{1}{\sqrt{1-s^2}}$$
$$= -\frac{s^2}{\sqrt{1-s^2}} + \sqrt{1-s^2} - \frac{1}{\sqrt{1-s^2}}$$
$$= -\frac{-s^2+(1-s^2)-1}{\sqrt{1-s^2}}$$
$$= -\frac{2s^2}{\sqrt{1-s^2}}$$

7. $$\frac{dy}{dx} = \frac{d}{dx}(x\sin^{-1} x) + \frac{d}{dx}(\sqrt{1-x^2})$$
$$= (x)\left(\frac{1}{\sqrt{1-x^2}}\right) + (\sin^{-1} x)(1) + \frac{1}{2\sqrt{1-x^2}}(-2x)$$
$$= \sin^{-1} x$$

8. $$\frac{dy}{dx} = \frac{d}{dx}[\sin^{-1}(2x)]^{-1}$$
$$= -[\sin^{-1}(2x)]^{-2}\frac{d}{dx}\sin^{-1}(2x)$$
$$= -[\sin^{-1}(2x)]^{-2}\frac{1}{\sqrt{1-4x^2}}(2)$$
$$= -\frac{2}{[\sin^{-1}(2x)]^2\sqrt{1-4x^2}}$$

9. $x(t) = \sin^{-1}\left(\dfrac{t}{4}\right)$

$$\frac{dx}{dt} = \frac{d}{dt}\left[\sin^{-1}\left(\frac{t}{4}\right)\right] = \frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}}\frac{d}{dt}\left(\frac{t}{4}\right)$$
$$= \frac{1}{\sqrt{1-\frac{t^2}{16}}}\cdot\frac{1}{4} = \frac{1}{\sqrt{16-t^2}}$$
$$v(3) = \left.\frac{dx}{dt}\right|_{t=3} = \frac{1}{\sqrt{16-3^2}} = \frac{\sqrt{7}}{7}$$

10. $$\frac{dx}{dt} = \frac{d}{dt}\left[\sin^{-1}\left(\frac{\sqrt{t}}{4}\right)\right] = \frac{1}{\sqrt{1-\left(\frac{\sqrt{t}}{4}\right)^2}}\frac{d}{dt}\left(\frac{\sqrt{t}}{4}\right)$$
$$= \frac{1}{\sqrt{1-\frac{t}{16}}}\cdot\frac{1}{8\sqrt{t}}$$
$$v(4) = \left.\frac{dx}{dt}\right|_{t=4} = \frac{1}{\sqrt{1-\frac{4}{16}}}\cdot\frac{1}{8\sqrt{4}}$$
$$= \frac{1}{\sqrt{1-\frac{1}{4}}}\cdot\frac{1}{16} = \frac{2}{\sqrt{3}}\cdot\frac{1}{16} = \frac{\sqrt{3}}{24}$$

11. $\frac{dx}{dt}=\frac{d}{dt}\left[\tan^{-1}t\right]=\frac{1}{1+t^2}$

$v(2)=\left.\frac{dx}{dt}\right|_{t=2}=\frac{1}{1+2^2}=\frac{1}{5}$

12. $\frac{dx}{dt}=\frac{d}{dt}\left[\tan^{-1}(t^2)\right]$

$=\frac{1}{1+(t^2)^2}\cdot\frac{d}{dt}(t^2)$

$=\frac{2t}{1+t^4}$

$v(1)=\left.\frac{dx}{dt}\right|_{t=1}=\frac{2(1)}{1+1^4}=1$

13. $\frac{dy}{ds}=\frac{d}{ds}\sec^{-1}(2s+1)$

$=\frac{1}{|2s+1|\sqrt{(2s+1)^2-1}}\frac{d}{ds}(2s+1)$

$=\frac{1}{|2s+1|\sqrt{4s^2+4s}}(2)=\frac{1}{|2s+1|\sqrt{s^2+s}}$

14. $\frac{dy}{ds}=\frac{d}{ds}\sec^{-1}5s=\frac{1}{|5s|\sqrt{(5s)^2-1}}\frac{d}{ds}(5s)=\frac{1}{|s|\sqrt{25s^2-1}}$

15. $\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}(x^2+1)$

$=-\frac{1}{|x^2+1|\sqrt{(x^2+1)^2-1}}\frac{d}{dx}(x^2+1)$

$=-\frac{2x}{(x^2+1)\sqrt{x^4+2x^2}}=-\frac{2}{(x^2+1)\sqrt{x^2+2}}$

Note that the condition $x>0$ is required in the last step.

16. $\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}\left(\frac{x}{2}\right)=-\frac{1}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2-1}}\frac{d}{dx}\left(\frac{x}{2}\right)$

$=-\frac{2}{|x|\sqrt{x^2-4}}$

17. $\frac{dy}{dt}=\frac{d}{dt}\sec^{-1}\left(\frac{1}{t}\right)=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\frac{d}{dt}\left(\frac{1}{t}\right)$

$=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\left(-\frac{1}{t^2}\right)=-\frac{1}{\sqrt{1-t^2}}$

Note that the condition $t>0$ is required in the last step.

18. $\frac{dy}{dt}=\frac{d}{dt}\cot^{-1}\sqrt{t}=-\frac{1}{1+(\sqrt{t})^2}\frac{d}{dt}\sqrt{t}$

$=-\frac{1}{2\sqrt{t}(t+1)}$

19. $\frac{dy}{dt}=\frac{d}{dt}\cot^{-1}\sqrt{t-1}=-\frac{1}{1+(\sqrt{t-1})^2}\frac{d}{dt}\sqrt{t-1}$

$=-\left(\frac{1}{1+t-1}\right)\left(\frac{1}{2\sqrt{t-1}}\right)=-\frac{1}{2t\sqrt{t-1}}$

20. $\frac{dy}{ds}=\frac{d}{ds}\sqrt{s^2-1}-\frac{d}{ds}\sec^{-1}s$

$=\frac{1}{2\sqrt{s^2-1}}(2s)-\frac{1}{|s|\sqrt{s^2-1}}$

$=\frac{s|s|-1}{|s|\sqrt{s^2-1}}$

21. $\frac{dy}{dx}=\frac{d}{dx}(\tan^{-1}\sqrt{x^2-1})+\frac{d}{dx}(\csc^{-1}x)$

$=\frac{1}{1+(\sqrt{x^2-1})^2}\frac{d}{dx}(\sqrt{x^2-1})-\frac{1}{|x|\sqrt{x^2-1}}$

$=\frac{1}{x^2}\frac{1}{2\sqrt{x^2-1}}(2x)-\frac{1}{|x|\sqrt{x^2-1}}$

$=\frac{1}{x\sqrt{x^2-1}}-\frac{1}{|x|\sqrt{x^2-1}}$

$=0$

Note that the condition $x>1$ is required in the last step.

22. $\frac{dy}{dx}=\frac{d}{dx}\left(\cot^{-1}\frac{1}{x}\right)-\frac{d}{dx}(\tan^{-1}x)$

$=-\frac{1}{1+\left(\frac{1}{x^2}\right)}\frac{d}{dx}\left(\frac{1}{x}\right)-\frac{1}{1+x^2}$

$=\left(-\frac{1}{1+\frac{1}{x^2}}\right)\left(-\frac{1}{x^2}\right)-\frac{1}{1+x^2}$

$=\frac{1}{x^2+1}-\frac{1}{1+x^2}$

$=0,\ x\neq 0$

The condition $x\neq 0$ is required because the original function was undefined when $x=0$.

23. $y=\sec^{-1}x$

$\frac{dy}{dx}=\frac{1}{|x|\sqrt{x^2-1}}$

$y'(2)=\frac{1}{|2|\sqrt{2^2-1}}=\frac{1}{2\sqrt{3}}$

$y(2)=\sec^{-1}(2)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

$y=\frac{1}{2\sqrt{3}}(x-2)+\frac{\pi}{3}$

or $y=0.289(x-2)+1.047$

$y=0.289x+0.469$

24. $y = \tan^{-1} x$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$y'(2) = \frac{1}{1+2^2} = \frac{1}{5}$$

$$y(2) = \tan^{-1}(2)$$

$$y = \frac{1}{5}(x-2) + \tan^{-1}(2)$$

or $y = 0.2(x-2) + 1.107$

$y = 0.2x + 0.707$

25. $y = \sin^{-1}\left(\frac{x}{4}\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}} \frac{d}{dx}\left(\frac{x}{4}\right)$$

$$= \frac{1}{\sqrt{1-\frac{x^2}{16}}} \bullet \frac{1}{4}$$

$$= \frac{1}{\sqrt{16-x^2}}$$

$$y'(3) = \frac{1}{\sqrt{16-3^2}} = \frac{1}{\sqrt{7}}$$

$$y(3) = \sin^{-1}\left(\frac{3}{4}\right)$$

$$y = \frac{1}{\sqrt{7}}(x-3) + \sin^{-1}\left(\frac{3}{4}\right)$$

or $y = 0.378(x-3) + 0.848$

$y = 0.378x - 0.286$

26. $y = \tan^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{1}{1+(x^2)^2} \bullet \frac{d}{dx}(x^2)$$

$$= \frac{1}{1+x^4} \bullet 2x$$

$$= \frac{2x}{1+x^4}$$

$$y'(1) = \frac{2(1)}{1+1^4} = \frac{2}{2} = 1$$

$$y(1) = \tan^{-1}(1^2)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$y = 1(x-1) + \frac{\pi}{4}$$

$$y = x - 1 + \frac{\pi}{4}$$

or $y = x - 0.215$

27. (a) Since $\frac{dy}{dx} = \sec^2 x$, the slope at $\left(\frac{\pi}{4}, 1\right)$ is $\sec^2\left(\frac{\pi}{4}\right) = 2$.

The tangent line is given by

$y = 2\left(x - \frac{\pi}{4}\right) + 1$, or $y = 2x - \frac{\pi}{2} + 1$.

(b) Since $\frac{dy}{dx} = \frac{1}{1+x^2}$, the slope at $\left(1, \frac{\pi}{4}\right)$ is $\frac{1}{1+1^2} = \frac{1}{2}$.

The tangent line is given by $y = \frac{1}{2}(x-1) + \frac{\pi}{4}$

or $y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}$

28. (a) Note that $f'(x) = 5x^4 + 6x^2 + 1$. Thus $f(1) = 3$ and $f'(1) = 12$.

(b) Since the graph of $y = f(x)$ includes the point (1, 3) and the slope of the graph is 12 at this point, the graph of $y = f^{-1}(x)$ will include (3, 1) and the slope will be $\frac{1}{12}$.

Thus, $f^{-1}(3) = 1$ and $(f^{-1})'(3) = \frac{1}{12}$. (We have assumed that $f^{-1}(x)$ is defined and differentiable at $x = 3$. This is true by Theorem 3, because $f'(x) = 5x^4 + 6x^2 + 1$, which is never zero.)

29. (a) Note that $f'(x) = -\sin x + 3$, which is always between 2 and 4. Thus f is differentiable at every point on the interval $(-\infty, \infty)$ and $f'(x)$ is never zero on this interval, so f has a differentiable inverse by Theorem 3.

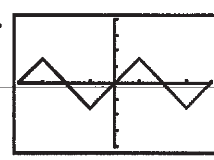
(b) $f(0) = \cos 0 + 3(0) = 1$;

$f'(0) = -\sin 0 + 3 = 3$

(c) Since the graph of $y = f(x)$ includes the point (0, 1) and the slope of the graph is 3 at this point, the graph of $y = f^{-1}(x)$ will include (1, 0) and the slope will be $\frac{1}{3}$,

Thus, $f^{-1}(1) = 0$ and $(f^{-1})'(1) = \frac{1}{3}$.

30.



[-2π, 2π] by [-4, 4]

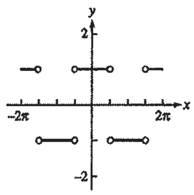
(a) All reals

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) At the points $x = k\frac{\pi}{2}$, where k is an odd integer.

30. Continued

(d)



(e) $f'(x) = \frac{d}{dx}\sin^{-1}(\sin x)$

$$= \frac{1}{\sqrt{1-\sin^2 x}}\frac{d}{dx}\sin x$$

$$= \frac{\cos x}{\sqrt{1-\sin^2 x}}$$

which is ± 1 depending on whether cos x is positive or negative.

31. (a) $v(t) = \frac{dx}{dt} = \frac{1}{1+t^2}$ which is always positive.

(b) $a(t) = \frac{dv}{dt} = -\frac{2t}{(1+t^2)^2}$ which is always negative.

(c) $\frac{\pi}{2}$

32. $\frac{d}{dx}\cos^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sin^{-1} x\right)$

$$= 0 - \frac{d}{dx}\sin^{-1}(x)$$

$$= -\frac{1}{\sqrt{1-x^2}}$$

33. $\frac{d}{dx}\cot^{-1} x = \frac{d}{dx}\left(\frac{\pi}{2} - \tan^{-1}(x)\right)$

$$= 0 - \frac{d}{dx}\tan^{-1}(x)$$

$$= -\frac{1}{1+x^2}$$

34. $\frac{d}{dx}\csc^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sec^{-1}(x)\right)$

$$= 0 - \frac{d}{dx}\sec^{-1}(x)$$

$$= -\frac{1}{|x|\sqrt{x^2-1}}$$

35. True. By definition of the function.

36. False. The domain is all real numbers.

37. E. $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{2}\right)$

$$= \frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2}$$

$$= \frac{1}{\sqrt{4-x^2}}$$

38. D. $\frac{d}{dx}\tan^{-1}(3x) = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x)$

$$= \frac{1}{1+9x^2} \cdot 3$$

$$= \frac{3}{1+9x^2}$$

39. A. $\frac{d}{dx}\sec^{-1}(x^2) = \frac{1}{|x^2|\sqrt{(x^2)^2-1}}\frac{d}{dx}(x^2)$

$$= \frac{1}{x^2\sqrt{x^4-1}} \cdot 2x$$

$$= \frac{2}{x\sqrt{x^4-1}}$$

40. C. $\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}(2x)\right)$

$$= \frac{1}{1+(2x)^2} \cdot \frac{d}{dx}(2x)$$

$$= \frac{1}{1+4x^2} \cdot 2$$

$$= \frac{2}{1+4x^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{2}{1+4(1)^2} = \frac{2}{5}$$

41. (a) $y = \frac{\pi}{2}$

(b) $y = -\frac{\pi}{2}$

(c) None, since $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2} \neq 0$.

42. (a) $y = 0$

(b) $y = \pi$

(c) None, since $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2} \neq 0$.

43. (a) $y = \frac{\pi}{2}$

(b) $y = \frac{\pi}{2}$

(c) None, since $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \neq 0$.

44. (a) $y = 0$

(b) $y = 0$

(c) None, since $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}} \neq 0$.

45. **(a)** None, since $\sin^{-1} x$ is undefined for $x > 1$.

(b) None, since $\sin^{-1} x$ is undefined for $x < -1$.

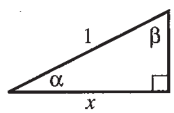
(c) None, since $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \neq 0$.

46. **(a)** None, since $\cos^{-1} x$ is undefined for $x > 1$.

(b) None, since $\cos^{-1} x$ is undefined for $x < -1$.

(c) None, since $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \neq 0$.

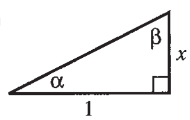
47. **(a)**



$\alpha = \cos^{-1} x, \beta = \sin^{-1} x$

Therefore, $\cos^{-1} x + \sin^{-1} x = \alpha + \beta = \frac{\pi}{2}$.

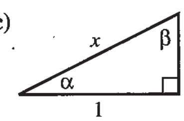
(b)



$\alpha = \tan^{-1} x, \beta = \cot^{-1} x$

Therefore, $\tan^{-1} x + \cot^{-1} x = \alpha + \beta = \frac{\pi}{2}$.

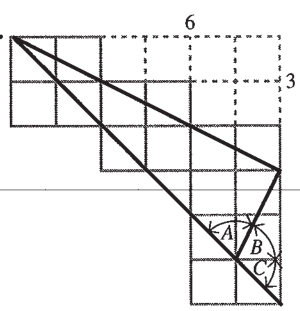
(c)



$\alpha = \sec^{-1} x, \beta = \csc^{-1} x$

Therefore, $\sec^{-1} x + \csc^{-1} x = \alpha + \beta = \frac{\pi}{2}$.

48.



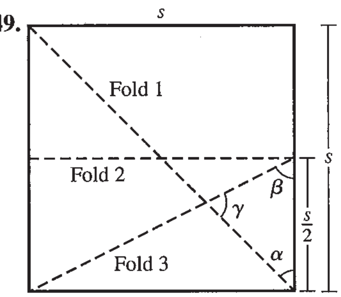
The "straight angle" with the arrows in it is the sum of the three angles A, B, and C.

A is equal to $\tan^{-1} 3$ since the opposite side is 3 times as long as the adjacent side.

B is equal to $\tan^{-1} 2$ since the side opposite it is 2 units and the adjacent side is one unit.

C is equal to $\tan^{-1} 1$ since both the opposite and adjacent sides are one unit long.
But the sum of these three angles is the "straight angle," which has measure π radians.

49.



If s is the length of a side of the square, then

$\tan \alpha = \frac{s}{s} = 1$, so $\alpha = \tan^{-1} 1$ and

$\tan \beta = \frac{s}{\frac{s}{2}} = 2$, so $\beta = \tan^{-1} 2$.

Then $\gamma = \pi - \alpha - \beta = \pi - \tan^{-1} 1 - \tan^{-1} 2 = \tan^{-1} 3$.

(In the last step, we used Exercise 48.)

Section 3.9 Derivatives of Exponential and Logarithmic Functions (pp. 172–180)

Exploration 1 Leaving Milk on the Counter

1. The temperature of the refrigerator is 42°F, the temperature of the milk at time $t = 0$.

2. The temperature of the room is 72°F, the limit to which y tends as t increases.

3. The milk is warming up the fastest at $t = 0$. The second derivative $y'' = -30(\ln(0.98))^2(0.98)^t$ is negative, so y' (the rate at which the milk is warming) is maximized at the lowest value of t.

4. We set $y = 55$ and solve;

$72 - 30(0.98)^t = 55$

$(0.98)^t = \frac{17}{30}$

$t\ln(0.98) = \ln\left(\frac{17}{30}\right)$

$t = \frac{\ln\left(\frac{17}{30}\right)}{\ln(0.98)} = 28.114$

The milk reaches a temperature of 55° F after about 28 minutes.

5. $\frac{dy}{dt} = -30\ln(0.98) \cdot (0.98)^t$. At $t = 28.114$,

$\frac{dy}{dt} \approx 0.343$ degrees/minute.

Quick Review 3.9

1. $\log_5 8 = \frac{\ln 8}{\ln 5}$

2. $7^x = e^{\ln 7^x} = e^{x \ln 7}$