46. The growth rate is given by

$b'(t) = 10^4 - 2 \cdot 10^3(t) = 10{,}000 - 2000t.$

At $t = 0$: $b'(0) = 10{,}000$ bacteria/hour

At $t = 5$: $b'(5) = 0$ bacteria/hour

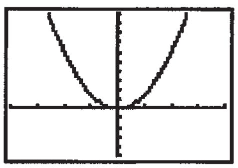
At $t = 10$: $b'(10) = -10{,}000$ bacteria/hour

47. (a) $g'(x) = \frac{d}{dx}(x^3) = 3x^2$

$h'(x) = \frac{d}{dx}(x^3 - 2) = 3x^2$

$t'(x) = \frac{d}{dx}(x^3 + 3) = 3x^2$

(b) The graphs of NDER $g(x)$, NDER $h(x)$, and NDER $t(x)$ are all the same, as shown.



[−4, 4] by [−10, 20]

(c) $f(x)$ must be of the form $f(x) = x^3 + c$, where c is a constant.

(d) Yes. $f(x) = x^3$

(e) Yes. $f(x) = x^3 + 3$

48. For $t > 0$, the speed of the aircraft in meters per second after t seconds is $v(t) = \frac{20}{9}t$. Multiplying by $\frac{3600 \text{ sec}}{1 \text{ h}} \cdot \frac{1 \text{ km}}{1000 \text{ m}}$, we find that this is equivalent to $8t$ kilometers per hour. Solving $8t = 200$ gives $t = 25$ seconds. The aircraft takes 25 seconds to become airborne, and the distance it travels during this time is $D(25) \approx 694.444$ meters.

49. (a) Assume that f is even. Then,

$$f'(-x) = \lim_{h\to 0}\frac{f(-x+h) - f(-x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x-h) - f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0}\frac{f(x+k) - f(x)}{-k}$$
$$= -\lim_{k\to 0}\frac{f(x+k) - f(x)}{k} = -f'(x)$$

So, f' is an odd function.

(b) Assume that f is odd. Then,

$$f'(-x) = \lim_{h\to 0}\frac{f(-x+h) - f(-x)}{h}$$
$$= \lim_{h\to 0}\frac{-f(x-h) + f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0}\frac{-f(x+k) + f(x)}{-k}$$
$$= \lim_{k\to 0}\frac{f(x+k) - f(x)}{k} = f'(x)$$

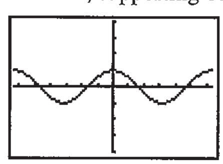
So, f' is an even function.

50. $$\frac{d}{dx}(fgh) = \frac{d}{dx}[f(gh)] = f \cdot \frac{d}{dx}(gh) + gh \cdot \frac{d}{dx}(f)$$
$$= f\left(g \cdot \frac{dh}{dx} + h \cdot \frac{dg}{dx}\right) + gh \cdot \frac{df}{dx}$$
$$= \left(\frac{df}{dx}\right)gh + f\left(\frac{dg}{dx}\right)h + fg\left(\frac{dh}{dx}\right)$$

Section 3.5 Derivatives of Trigonometric Functions (pp. 141–147)

Exploration 1 Making a Conjecture with NDER

1. When the graph of sin x is increasing, the graph of NDER (sin x) is positive (above the x-axis).
2. When the graph of sin x is decreasing, the graph of NDER (sin x) is negative (below the x-axis).
3. When the graph of sin x stops increasing and starts decreasing, the graph of NDER (sin x) crosses the x-axis from above to below.
4. The slope of the graph of sin x matches the value of NDER (sin x) at these points.
5. We conjecture that NDER (sin x) = cos x. The graphs coincide, supporting our conjecture.

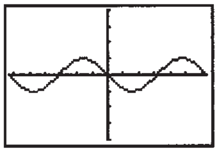


[−2π, 2π] by [−4, 4]

6. When the graph of cos x is increasing, the graph of NDER (cos x) is positive (above the x-axis).
When the graph of cos x is decreasing, the graph of NDER (cos x) is negative (below the x-axis).
When the graph of cos x stops increasing and starts decreasing, the graph of NDER (cos x) crosses the x-axis from above to below.
The slope of the graph of cos x matches the value of NDER (cos x) at these points.

6. Continued

We conjecture that NDER $(\cos x) = -\sin x$. The graphs coincide, supporting our conjecture.



$[-2\pi, 2\pi]$ by $[-4, 4]$

Quick Review 3.5

1. $135° \cdot \frac{\pi}{180°} = \frac{3\pi}{4} \approx 2.356$

2. $1.7 \cdot \frac{180°}{\pi} = \left(\frac{306}{\pi}\right)° \approx 97.403°$

3. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

4. Domain: All reals
Range: $[-1, 1]$

5. Domain: $x \neq \frac{k\pi}{2}$ for odd integers k
Range: All reals

6. $\cos a = \pm\sqrt{1-\sin^2 a} = \pm\sqrt{1-(-1)^2} = \pm\sqrt{0} = 0$

7. If $\tan a = -1$, then $a = \frac{3\pi}{4} + k\pi$ for some integer k,

so $\sin a = \pm\frac{1}{\sqrt{2}}$.

8. $\frac{1-\cos h}{h} = \frac{(1-\cos h)(1+\cos h)}{h(1+\cos h)} = \frac{1-\cos^2 h}{h(1+\cos h)}$

$= \frac{\sin^2 h}{h(1+\cos h)}$

9. $y'(x) = 6x^2 - 14x$
$y'(3) = 12$
The tangent line has slope 12 and passes through (3, 1), so its equation is $y = 12(x-3)+1$, or $y = 12x - 35$.

10. $a(t) = v'(t) = 6t^2 - 14t$
$a(3) = 12$

Section 3.5 Exercises

1. $\frac{d}{dx}(1+x-\cos x) = 0+1-(-\sin x) = 1+\sin x$

2. $\frac{d}{dx}(2\sin x - \tan x) = 2\cos x - \sec^2 x$

3. $\frac{d}{dx}\left(\frac{1}{x}+5\sin x\right) = -\frac{1}{x^2}+5\cos x$

4. $\frac{d}{dx}(x\sec x) = x\frac{d}{dx}(\sec x)+\sec x\frac{d}{dx}(x)$
$= x\sec x\tan x + \sec x$

5. $\frac{d}{dx}(4-x^2\sin x) = \frac{d}{dx}(4) - \left[x^2\frac{d}{dx}(\sin x)+(\sin x)\frac{d}{dx}(x^2)\right]$
$= 0-[x^2\cos x + (\sin x)(2x)]$
$= -x^2\cos x - 2x\sin x$

6. $\frac{d}{dx}(3x+x\tan x) = \frac{d}{dx}(3x)+\left[x\frac{d}{dx}(\tan x)+(\tan x)\frac{d}{dx}(x)\right]$
$= 3+x\sec^2 x+\tan x$

7. $\frac{d}{dx}\left(\frac{4}{\cos x}\right) = \frac{d}{dx}(4\sec x) = 4\sec x\tan x$

8. $\frac{d}{dx}\left(\frac{x}{1+\cos x}\right) = \frac{(1+\cos x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$
$= \frac{1+\cos x+x\sin x}{(1+\cos x)^2}$

9. $\frac{d}{dx}\left(\frac{\cot x}{1+\cot x}\right) = \frac{(1+\cot x)\frac{d}{dx}(\cot x) - (\cot x)\frac{d}{dx}(1+\cot x)}{(1+\cot x)^2}$
$= \frac{(1+\cot x)(-\csc^2 x)-(\cot x)(-\csc^2 x)}{(1+\cot x)^2}$
$= -\frac{\csc^2 x}{(1+\cot x)^2} = -\frac{\csc^2 x\sin^2 x}{(1+\cot x)^2\sin^2 x} = -\frac{1}{(\sin x+\cos x)^2}$

10. $\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right) = \frac{(1+\sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$
$= \frac{(1+\sin x)(-\sin x)-(\cos x)(\cos x)}{(1+\sin x)^2}$
$= \frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$
$= \frac{-(1+\sin x)}{(1+\sin x)^2}$
$= -\frac{1}{1+\sin x}$

11. $v(t) = \frac{ds}{dt} = \frac{d}{dx}(5\sin t)$
$v(t) = 5\cos t$
$a(t) = \frac{dv}{dt} = \frac{d}{dx}(5\cos t)$
$a(t) = -5\sin t$
The weight starts at 0, goes to 5, and the oscillates between 5 and –5. The period of the motion is 2π. The speed is greatest when $\cos t = \pm 1$ $(t = k\pi)$, zero when $\cos t = 0$ $\left(t = \frac{k\pi}{2}, k \text{ odd}\right)$. The acceleration is greatest when $\sin t = \pm 1$ $\left(t = \frac{k\pi}{2}, k \text{ odd}\right)$, zero when $\sin t = 0$ $(t = k\pi)$.

12. $v(t)=\frac{ds}{dt}=\frac{d}{dx}(7\cos t)$
$v(t)=-7\sin t$
$a(t)=\frac{dv}{dt}=\frac{d}{dx}(-7\sin t)$
$a(t)=-7\cos t$

The weight starts at 7, goes to −7, and then oscillates between −7 and 7. The period of the motion is 2π. The speed is greatest when $\sin t=\pm 1\left(t=\frac{k\pi}{2},\ k\text{ odd}\right)$, zero when $\sin t=0$ $(t=k\pi)$. The acceleration is greatest when $\cos t=\pm 1$ $(t=k\pi)$, zero when $\cos t=0\left(t=\frac{k\pi}{2},\ k\text{ odd}\right)$.

13. **(a)** $v(t)=\frac{ds}{dt}=\frac{d}{dt}(2+3\sin t)$
$v(t)=3\cos t$, speed $=|3\cos t|$
$a(t)=\frac{dv}{dt}=\frac{d}{dt}(3\cos t)=-3\sin t$

(b) $v\left(\frac{\pi}{4}\right)=3\cos\left(\frac{\pi}{4}\right)=\frac{3\sqrt{2}}{2}$, speed $=\frac{3\sqrt{2}}{2}$
$a\left(\frac{\pi}{4}\right)=-3\sin\left(\frac{\pi}{4}\right)=-\frac{3\sqrt{2}}{2}$

(c) The body starts at 2, goes up to 5, goes down to −1, and then oscillates between −1 and 5. The period of motion is 2π.

14. **(a)** $v(t)=\frac{ds}{dt}=\frac{d}{dt}(1-4\cos t)$
$v(t)=4\sin t$, speed $=|4\sin t|$
$a(t)=\frac{dv}{dt}=\frac{d}{dt}(4\sin t)$
$a(t)=4\cos t$

(b) $v\left(\frac{\pi}{4}\right)=4\sin\left(\frac{\pi}{4}\right)=2\sqrt{2}$, speed $=2\sqrt{2}$
$a\left(\frac{\pi}{4}\right)=4\cos\left(\frac{\pi}{4}\right)=2\sqrt{2}$

(c) The body starts at −3, goes up to 5, and then oscillates between 5 and −3. The period of the motion is 2π.

15. **(a)** $v(t)=\frac{ds}{dt}=\frac{d}{dt}(2\sin t+3\cos t)$
$v(t)=2\cos t-3\sin t$, speed $=|2\cos t-3\sin t|$
$a(t)=\frac{dv}{dt}=\frac{d}{dt}(2\cos t-3\sin t)$
$a(t)=-2\sin t-3\cos t$

(b) $v\left(\frac{\pi}{4}\right)=2\cos\frac{\pi}{4}-3\sin\frac{\pi}{4}$
$v\left(\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$
speed $=\frac{\sqrt{2}}{2}$
$a\left(\frac{\pi}{4}\right)=-2\sin\frac{\pi}{4}-3\cos\frac{\pi}{4}$
$a\left(\frac{\pi}{4}\right)=\frac{-5\sqrt{2}}{2}$

(c) The body starts at 3, goes to 3.606, and then oscillates between −3.606 and 3.606. The period of the motion is 2π.

16. **(a)** $v(t)=\frac{ds}{dt}=\frac{d}{dt}(\cos t-3\sin t)$
$v(t)=-\sin t-3\cos t$, speed $=|-\sin t-3\cos t|$
$a(t)=\frac{dv}{dt}=\frac{d}{dt}(-\sin t-3\cos t)$
$a(t)=-\cos t+3\sin t$

(b) $v\left(\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}-3\cos\frac{\pi}{4}=-2\sqrt{2}$
speed $=2\sqrt{2}$
$a\left(\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}+3\sin\frac{\pi}{4}=\sqrt{2}$

(c) The body starts at 1, goes to −3.162, and then oscillates between 3.162 and −3.162. The period of the motion is 2π.

17. $j(t)=\frac{da}{dt}=\frac{d^3s}{dt^3}$
$f(t)=2\cos t$
$f'(t)=-2\sin t$
$f''(t)=-2\cos t$
$f'''(t)=2\sin t$

18. $j(t)=\frac{da}{dt}=\frac{d^3s}{dt^3}$
$f(t)=1+2\cos t$
$f'(t)=-2\sin t$
$f''(t)=-2\cos t$
$f'''(t)=2\sin t$

19. $j(t)=\frac{da}{dt}=\frac{d^3s}{dt^3}$
$f(t)=\sin t-\cos t$
$f'(t)=\cos t+\sin t$
$f''(t)=-\sin t+\cos t$
$f'''(t)=-\cos t-\sin t$

20. $j(t) = \frac{da}{dt} = \frac{d^3s}{dt}$

$f(t) = 2 + 2\sin t$
$f'(t) = 2\cos t$
$f''(t) = -2\sin t$
$f'''(t) = -2\cos t$

21. $y = \sin x + 3$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x + 3) = \cos x$
$y(\pi) = \sin\pi + 3 = 3$
$y'(\pi) = \cos\pi = -1$
tangent: $y = -1(x-\pi) + 3 = -x + \pi + 3$
normal: $m_2 = -\frac{1}{m_1} = 1$
$y = (x - \pi) + 3$

22. $y = \sec x$

$\frac{dy}{dx} = \frac{d}{dx}(\sec x) = \sec x \tan x$
$y\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4} = 1.414$
$y'\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4}\tan\frac{\pi}{4} = 1.414$
tangent: $y = 1.414(x - \frac{\pi}{4}) + 1.414$
$y = 1.414x + 0.303$
normal: $m_2 = -\frac{1}{m_1} = -0.707$
$y = -0.707(x - \frac{\pi}{4}) + 1.414 = -0.707x + 1.970$

23. $y = x^2 \sin x$

$\frac{dy}{dx} = \frac{d}{dx}(x^2\sin x) = 2x\sin x + x^2\cos x$
$y(3) = (3)^2 \sin 3 = 1.270$
$y'(3) = 2(3)\sin 3 + (3)^2\cos 3 = -8.063$
tangent: $y = -8.063(x-3) + 1.270 = -8.063x + 25.460$
normal: $m_2 = -\frac{1}{m_1} = 0.124$
$y = 0.124(x-3) + 1.270$
$y = 0.124x + 0.898$

24. $\lim_{h\to 0}\frac{\cos(x+h) - \cos x}{h}$

$= \lim_{h\to 0}\frac{(\cos x\cos h - \sin x\sin h) - \cos x}{h}$
$= \lim_{h\to 0}\frac{\cos x(\cos h - 1) - \sin x\sin h}{h}$
$= \lim_{h\to 0}\left((\cos x)\frac{\cos h - 1}{h} - (\sin x)\frac{\sin h}{h}\right)$
$= (\cos x)\left(\lim_{h\to 0}\frac{\cos h - 1}{h}\right) - (\sin x)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$
$= (\cos x)(0) - (\sin x)(1) = -\sin x$

25. (a) $\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{(\cos x)\frac{d}{dx}(\sin x) - (\sin x)\frac{d}{dx}(\cos x)}{(\cos x)^2}$

$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$
$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

(b) $\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{(\cos x)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\cos x)}{(\cos x)^2}$

$= \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$
$= \frac{\sin x}{\cos^2 x} = \sec x \tan x$

26. (a) $\frac{d}{dx}\cot x = \frac{d}{dx}\frac{\cos x}{\sin x}$

$= \frac{(\sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(\sin x)}{(\sin x)^2}$
$= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$
$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$
$= -\frac{1}{\sin^2 x} = -\csc^2 x$

(b) $\frac{d}{dx}\csc x = \frac{d}{dx}\frac{1}{\sin x}$

$= \frac{(\sin x)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\sin x)}{(\sin x)^2}$
$= \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x}$
$= -\frac{\cos x}{\sin^2 x} = -\csc x \cot x$

27. $\frac{d}{dx}\sec x = \sec x\tan x$ which is 0 at $x = 0$, so the slope of the tangent line is 0. $\frac{d}{dx}\cos x = -\sin x$ which is 0 at $x = 0$, so the slope of the tangent line is 0.

28. $\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$, which is never 0.

$\frac{d}{dx}\cot x = -\csc^2 x = -\frac{1}{\sin^2 x}$, which is never 0.

29. $y'(x) = \frac{d}{dx}(\sqrt{2}\cos x) = -\sqrt{2}\sin x$

$y'\left(\frac{\pi}{4}\right) = -\sqrt{2}\sin\frac{\pi}{4} = -\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -1$

The tangent line has slope -1 and passes through $\left(\frac{\pi}{4}, \sqrt{2}\cos\frac{\pi}{4}\right) = \left(\frac{\pi}{4}, 1\right)$, so its equation is

$y = -1\left(x - \frac{\pi}{4}\right) + 1$, or $y = -x + \frac{\pi}{4} + 1$.

The normal line has slope 1 and passes through $\left(\frac{\pi}{4}, 1\right)$, so its equation is $y = 1\left(x - \frac{\pi}{4}\right) + 1$, or $y = x + 1 - \frac{\pi}{4}$.

30. $y'(x) = \frac{d}{dx}\tan x = \sec^2 x$

$y'(x) = \frac{d}{dx}(2x) = 2$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

On $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the solutions are $x = \pm\frac{\pi}{4}$. The points on the curve are $\left(-\frac{\pi}{4}, -1\right)$ and $\left(\frac{\pi}{4}, 1\right)$.

31. $y'(x) = \frac{d}{dx}(4 + \cot x - 2\csc x)$

$= 0 - \csc^2 x + 2\csc x\cot x$

$= -\csc^2 x + 2\csc x\cot x$

(a) $y'\left(\frac{\pi}{2}\right) = -\csc^2\frac{\pi}{2} + 2\csc\frac{\pi}{2}\cot\frac{\pi}{2}$

$= -1^2 + 2(1)(0) = -1$

The tangent line has slope -1 and passes through $P\left(\frac{\pi}{2}, 2\right)$. Its equation is $y = -1\left(x - \frac{\pi}{2}\right) + 2$, or

$y = -x + \frac{\pi}{2} + 2$.

(b)
$$f'(x) = 0$$
$$-\csc^2 x + 2\csc x\cot x = 0$$
$$-\frac{1}{\sin^2 x} + \frac{2\cos x}{\sin^2 x} = 0$$
$$\frac{1}{\sin^2 x}(2\cos x - 1) = 0$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} \text{ at point } Q$$

$y\left(\frac{\pi}{3}\right) = 4 + \cot\frac{\pi}{3} - 2\csc\frac{\pi}{3}$

$= 4 + \frac{1}{\sqrt{3}} - 2\left(\frac{2}{\sqrt{3}}\right)$

$= 4 - \frac{3}{\sqrt{3}} = 4 - \sqrt{3}$

The coordinates of Q are $\left(\frac{\pi}{3}, 4 - \sqrt{3}\right)$.

The equation of the horizontal line is $y = 4 - \sqrt{3}$.

32. $y'(x) = \frac{d}{dx}(1 + \sqrt{2}\csc x + \cot x)$

$= 0 + \sqrt{2}(-\csc x\cot x) + (-\csc^2 x)$

$= -\sqrt{2}\csc x\cot x - \csc^2 x$

(a) $y'\left(\frac{\pi}{4}\right) = -\sqrt{2}\csc\frac{\pi}{4}\cot\frac{\pi}{4} - \csc^2\frac{\pi}{4}$

$= -\sqrt{2}(\sqrt{2})(1) - (\sqrt{2})^2$

$= -2 - 2 = -4$

The tangent line has slope -4 and passes through $P\left(\frac{\pi}{4}, 4\right)$. Its equation is $y = -4\left(x - \frac{\pi}{4}\right) + 4$, or

$y = -4x + \pi + 4$.

(b)
$$y'(x) = 0$$
$$-\sqrt{2}\csc x\cot x - \csc^2 x = 0$$
$$-\frac{\sqrt{2}\cos x}{\sin^2 x} - \frac{1}{\sin^2 x} = 0$$
$$-\frac{1}{\sin^2 x}(\sqrt{2}\cos x + 1) = 0$$
$$\cos x = -\frac{1}{\sqrt{2}}$$
$$x = \frac{3\pi}{4} \text{ at point } Q$$

$y\left(\frac{3\pi}{4}\right) = 1 + \sqrt{2}\csc\frac{3\pi}{4} + \cot\frac{3\pi}{4}$

$= 1 + \sqrt{2}(\sqrt{2}) + (-1)$

$= 2$

The coordinates of Q are $\left(\frac{3\pi}{4}, 2\right)$.

The equation of the horizontal line is $y = 2$.

33. **(a)** Velocity: $s'(t) = -2\cos t$ m/sec

Speed: $|s'(t)| = |2\cos t|$ m/sec

Acceleration: $s''(t) = 2\sin t$ m/sec^2

Jerk: $s'''(t) = 2\cos t$ m/sec^3

(b) Velocity: $-2\cos\frac{\pi}{4} = -\sqrt{2}$ m/sec

Speed: $|-\sqrt{2}| = \sqrt{2}$ m/sec

Acceleration: $2\sin\frac{\pi}{4} = \sqrt{2}$ m/sec^2

Jerk: $2\cos\frac{\pi}{4} = \sqrt{2}$ m/sec^3

(c) The body starts at 2, goes to 0 and then oscillates between 0 and 4.

Speed:

Greatest when $\cos t = \pm 1$ (or $t = k\pi$), at the center of the interval of motion.

Zero when $\cos t = 0$ (or $t = \frac{k\pi}{2}$, k odd), at the endpoints of the interval of motion.

Acceleration:

Greatest (in magnitude) when $\sin t = \pm 1$ (or $t = \frac{k\pi}{2}$, k odd)

Zero when $\sin t = 0$ (or $t = k\pi$)

Jerk:

Greatest (in magnitude) when $\cos t = \pm 1$ (or $t = k\pi$).

Zero when $\cos t = 0$ (or $t = \frac{k\pi}{2}$, k odd)

34. **(a)** Velocity: $s'(t) = \cos t - \sin t$ m/sec

Speed: $s'(t) = |\cos t - \sin t|$ m/sec

Acceleration: $s''(t) = -\sin t - \cos t$ m/sec^2

Jerk: $s'''(t) = -\cos t + \sin t$ m/sec^3

(b) Velocity: $\cos\frac{\pi}{4} - \sin\frac{\pi}{4} = 0$ m/sec

Speed: $|0| = 0$ m/sec

Acceleration: $-\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\sqrt{2}$ m/sec^2

Jerk: $-\cos\frac{\pi}{4} + \sin\frac{\pi}{4} = 0$ m/sec^3

(c) The body starts at 1, goes to $\sqrt{2}$ and then oscillates between $\pm\sqrt{2}$.

Speed:

Greatest when $t = \frac{3\pi}{4} + k\pi$

Zero when $t = \frac{\pi}{4} + k\pi$

Acceleration:

Greatest (in magnitude) when $t = \frac{\pi}{4} + k\pi$

Zero when $t = \frac{3\pi}{4} + k\pi$

Jerk:

Greatest (in magnitude) when $t = \frac{3\pi}{4} + k\pi$

Zero when $t = \frac{\pi}{4} + k\pi$

35. $$y' = \frac{d}{dx}\csc x = -\csc x \cot x$$

$$y'' = \frac{d}{dx}(-\csc x \cot x)$$
$$= -(\csc x)\frac{d}{dx}(\cot x) - (\cot x)\frac{d}{dx}(\csc x)$$
$$= -(\csc x)(-\csc^2 x) - (\cot x)(-\csc x \cot x)$$
$$= \csc^3 x + \csc x \cot^2 x$$

36. $$y' = \frac{d}{d\theta}(\theta \tan\theta)$$
$$= \theta\frac{d}{d\theta}(\tan\theta) + (\tan\theta)\frac{d}{d\theta}(\theta)$$
$$= \theta\sec^2\theta + \tan\theta$$

$$y'' = \frac{d}{d\theta}(\theta\sec^2\theta + \tan\theta)$$
$$= \theta\frac{d}{d\theta}[(\sec\theta)(\sec\theta)] + (\sec^2\theta)\frac{d}{d\theta}(\theta) + \frac{d}{d\theta}(\tan\theta)$$
$$= \theta\left[(\sec\theta)\frac{d}{d\theta}(\sec\theta) + (\sec\theta)\frac{d}{d\theta}(\sec\theta)\right] + \sec^2\theta + \sec^2\theta$$
$$= 2\theta\sec^2\theta\tan\theta + 2\sec^2\theta$$
$$= (2\theta\tan\theta + 2)(\sec^2\theta)$$

or, writing in terms of sines and cosines,

$$= \frac{2 + 2\theta\tan\theta}{\cos^2\theta}$$
$$= \frac{2\cos\theta + 2\theta\sin\theta}{\cos^3\theta}$$

37. Continuous:

Note that $g(0) = \lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \cos x = \cos(0) = 1$, and $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}(x+b) = b$. We require $\lim_{x\to 0^-} g(x) = g(0)$, so $b = 1$. The function is continuous if $b = 1$.

Differentiable:

For $b = 1$, the left-hand derivative is 1 and the right-hand derivative is $-\sin(0) = 0$, so the function is not differentiable. For other values of b, the function is discontinuous at $x = 0$ and there is no left-hand derivative. So, there is no value of b that will make the function differentiable at $x = 0$.

38. Observe the pattern:

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d^5}{dx^5}\cos x = -\sin x$$
$$\frac{d^2}{dx^2}\cos x = -\cos x \qquad \frac{d^6}{dx^6}\cos x = -\cos x$$
$$\frac{d^3}{dx^3}\cos x = \sin x \qquad \frac{d^7}{dx^7}\cos x = \sin x$$
$$\frac{d^4}{dx^4}\cos x = \cos x \qquad \frac{d^8}{dx^8}\cos x = \cos x$$

Continuing the pattern, we see that

$\frac{d^n}{dx^n}\cos x = \sin x$ when $n = 4k + 3$ for any whole number k.

Since $999 = 4(249) + 3$, $\frac{d^{999}}{dx^{999}}\cos x = \sin x$.

39. Observe the pattern:

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d^5}{dx^5}\sin x = \cos x$$
$$\frac{d^2}{dx^2}\sin x = -\sin x \qquad \frac{d^6}{dx^6}\sin x = -\sin x$$
$$\frac{d^3}{dx^3}\sin x = -\cos x \qquad \frac{d^7}{dx^7}\sin x = -\cos x$$
$$\frac{d^4}{dx^4}\sin x = \sin x \qquad \frac{d^8}{dx^8}\sin x = \sin x$$

Continuing the pattern, we see that

$\frac{d^n}{dx^n}\sin x = \cos x$ when $n = 4k+1$ for any whole number k.

Since $725 = 4(181) + 1$, $\frac{d^{725}}{dx^{725}}\sin x = \cos x$.

40. The line is tangent to the graph of $y = \sin x$ at (0, 0). Since $y'(0) = \cos(0) = 1$, the line has slope 1 and its equation is $y = x$.

41. (a) Using $y = x$, $\sin(0.12) \approx 0.12$.

(b) $\sin(0.12) \approx 0.1197122$; The approximation is within 0.0003 of the actual value.

42.
$$\frac{d}{dx}\sin 2x = \frac{d}{dx}(2\sin x\cos x)$$
$$= 2\frac{d}{dx}(\sin x\cos x)$$
$$= 2\left[(\sin x)\frac{d}{dx}(\cos x) + (\cos x)\frac{d}{dx}(\sin x)\right]$$
$$= 2[(\sin x)(-\sin x) + (\cos x)(\cos x)]$$
$$= 2(\cos^2 x - \sin^2 x)$$
$$= 2\cos 2x$$

43.
$$\frac{d}{dx}\cos 2x = \frac{d}{dx}[(\cos x)(\cos x) - (\sin x)(\sin x)]$$
$$= \left[(\cos x)\frac{d}{dx}(\cos x) + (\cos x)\frac{d}{dx}(\cos x)\right] - \left[(\sin x)\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(\sin x)\right]$$
$$= 2(\cos x)(-\sin x) - 2(\sin x)(\cos x)$$
$$= -4\sin x\cos x$$
$$= -2(2\sin x\cos x)$$
$$= -2\sin 2x$$

44. True. $s'(t) = -3\cos t$, $s'\left(\frac{3\pi}{4}\right) = -3\cos\left(\frac{3\pi}{4}\right) = \frac{3\sqrt{2}}{2} > 0$.

The derivative is positive at $t = \frac{3\pi}{4}$.

45. False. The velocity is negative and the speed is positive at $t = \frac{\pi}{4}$.

46. A.
$$y = \sin x + \cos x$$
$$y'(x) = \cos x - \sin x$$
$$y(\pi) = \sin\pi + \cos\pi = -1$$
$$y'(\pi) = \cos\pi - \sin\pi = -1$$
$$y = -1(x-\pi) - 1$$
$$y = -x + \pi - 1$$

47. B. See 46.
$$m_2 = -\frac{1}{m_1} = -\frac{1}{-1} = 1$$
$$y = (x-\pi) - 1$$

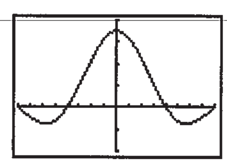
48. C.
$$y = x\sin x$$
$$y' = \sin x + x\cos x$$
$$y'' = \cos x + \cos x - x\sin x$$
$$= -x\sin x + 2\cos x$$

49. C. $v(t) = \frac{ds}{dt} = \frac{d}{dt}(3 + \sin t)$

$v(t) = \cos t = 0$

$t = \frac{\pi}{2}$

50. (a)

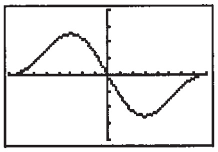


[–360, 360] by [–0.01, 0.02]

The limit is $\frac{\pi}{180}$ because this is the conversion factor for changing from degrees to radians.

50. Continued

(b)



[–360, 360] by [–0.02, 0.02]

This limit is still 0.

(c) $\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

$= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

$= \lim_{h\to 0}\frac{\sin x(\cos h-1)+\cos x\sin h}{h}$

$= \left(\lim_{h\to 0}\sin x\right)\left(\lim_{h\to 0}\frac{\cos h-1}{h}\right)+\left(\lim_{h\to 0}\cos x\right)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$

$= (\sin x)(0)+(\cos x)\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{180}\cos x$

(d) $\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$

$= \lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}$

$= \lim_{h\to 0}\frac{(\cos x)(\cos h-1)-\sin x\sin h}{h}$

$= \left(\lim_{h\to 0}\cos x\right)\left(\lim_{h\to 0}\frac{\cos h-1}{h}\right)-\left(\lim_{h\to 0}\sin x\right)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$

$= (\cos x)(0)-(\sin x)\left(\frac{\pi}{180}\right)$

$= -\frac{\pi}{180}\sin x$

(e) $\frac{d^2}{dx^2}\sin x = \frac{d}{dx}\frac{\pi}{180}\cos x = \frac{\pi}{180}\left(-\frac{\pi}{180}\sin x\right)$

$= -\frac{\pi^2}{180^2}\sin x$

$\frac{d^3}{dx^3}\sin x = \frac{d}{dx}\left(-\frac{\pi^2}{180^2}\sin x\right) = -\frac{\pi^2}{180^2}\left(\frac{\pi}{180}\cos x\right)$

$= -\frac{\pi^3}{180^3}\cos x$

$\frac{d^2}{dx^2}\cos x = \frac{d}{dx}\left(-\frac{\pi}{180}\sin x\right) = -\frac{\pi}{180}\left(\frac{\pi}{180}\cos x\right)$

$= -\frac{\pi^2}{180^2}\cos x$

$\frac{d^3}{dx^3}\cos x = \frac{d}{dx}\left(-\frac{\pi^2}{180^2}\cos x\right) = -\frac{\pi^2}{180^2}\left(-\frac{\pi}{180}\sin x\right)$

$= \frac{\pi^3}{180^3}\sin x$

51. $\lim_{h\to 0}\frac{(\cos h-1)}{h} = \lim_{h\to 0}\frac{(\cos h-1)(\cos h+1)}{h(\cos h+1)}$

$= \lim_{h\to 0}\frac{\cos^2 h-1}{h(\cos h+1)}$

$= \lim_{h\to 0}\frac{-\sin^2 h}{h(\cos h+1)}$

$= -\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\left(\lim_{h\to 0}\frac{\sin h}{\cos h+1}\right)$

$= -(1)\left(\frac{0}{2}\right) = 0$

52. $y' = \frac{d}{dx}(A\sin x+B\cos x) = A\cos x-B\sin x$

$y'' = \frac{d}{dx}(A\cos x-B\sin x) = -A\sin x-B\cos x$

Solve: $y''-y = \sin x$

$(-A\sin x-B\cos x)-(A\sin x+B\cos x) = \sin x$

$-2A\sin x-2B\cos x = \sin x$

At $x = \frac{\pi}{2}$, this gives $-2A = 1$, so $A = -\frac{1}{2}$.

At $x = 0$, we have $-2B = 0$, so $B = 0$.

Thus, $A = -\frac{1}{2}$ and $B = 0$.

Section 3.6 Chain Rule (pp. 148–156)

Quick Review 3.6

1. $f(g(x)) = f(x^2+1) = \sin(x^2+1)$

2. $f(g(h(x))) = f(g(7x)) = f((7x)^2+1)$
   $= \sin[(7x)^2+1] = \sin(49x^2+1)$

3. $(g\circ h)(x) = g(h(x)) = g(7x) = (7x)^2+1 = 49x^2+1$

4. $(h\circ g)(x) = h(g(x)) = h(x^2+1) = 7(x^2+1) = 7x^2+7$

5. $f\left(\frac{g(x)}{h(x)}\right) = f\left(\frac{x^2+1}{7x}\right) = \sin\frac{x^2+1}{7x}$

6. $\sqrt{\cos x+2} = g(\cos x) = g(f(x))$

7. $\sqrt{3\cos^2 x+2} = g(3\cos^2 x) = g(h(\cos x)) = g(h(f(x)))$

8. $3\cos x+6 = 3(\cos x+2) = 3(\sqrt{\cos x+2})^2$
   $= h(\sqrt{\cos x+2}) = h(g(\cos x)) = h(g(f(x)))$

9. $\cos 27x^4 = f(27x^4) = f(3(3x^2)^2) = f(h(3x^2)) = f(h(h(x)))$

10. $\cos\sqrt{2+3x^2} = \cos\sqrt{3x^2+2} = f(\sqrt{3x^2+2})$
    $= f(g(3x^2)) = f(g(h(x)))$