4. Continued

(c) $m_2 = -\frac{1}{m_1} = \frac{1}{4}$

$y = \frac{1}{4}(x-1) - 3$

$= \frac{1}{4}x - \frac{1}{4} - 3 = \frac{1}{4}x - \frac{13}{4}$

Section 3.4 Velocity and Other Rates of Change (pp. 127–140)

Exploration 1 Growth Rings on a Tree

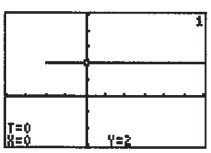
1. Figure 3.22 is a better model, as it shows rings of equal *area* as opposed to rings of equal *width*. It is not likely that a tree could sustain increased growth year after year, although climate conditions do produce some years of greater growth than others.

2. Rings of equal area suggest that the tree adds approximately the same amount of wood to its girth each year. With access to approximately the same raw materials from which to make the wood each year, this is how most trees actually grow.

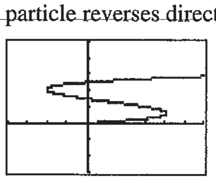
3. Since change in area is constant, so also is $\frac{\text{change in area}}{2\pi}$.
If we denote this latter constant by k, we have
$\frac{k}{\text{change in radius}} = r$, which means that r varies inversely as the change in the radius. In other words, the change in radius must get smaller when r gets bigger, and vice-versa.

Exploration 2 Modeling Horizontal Motion

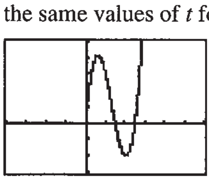
1. The particle reverses direction at about $t = 0.61$ and $t = 2.06$.



2. When the trace cursor is moving to the right the particle is moving to the right, and when the cursor is moving to the left the particle is moving to the left. Again we find the particle reverses direction at about $t = 0.61$ and $t = 2.06$.



3. When the trace cursor is moving upward the particle is moving to the right, and when the cursor is moving downward the particle is moving to the left. Again we find the same values of t for when the particle reverses direction.

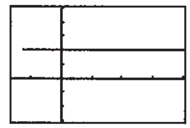


4. We can represent the velocity by graphing the parametric equations

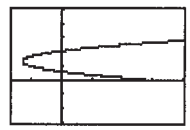
$x_4(t) = x_1'(t) = 12t^2 - 32t + 15,\ y_4(t) = 2$ (part 1),

$x_5(t) = x_1'(t) = 12t^2 - 32t + 15,\ y_5(t) = t$ (part 2),

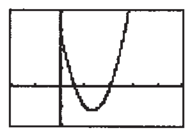
$x_6(t) = t,\ y_6(t) = x_1'(t) = 12t^2 - 32t + 15$ (part 3)



[−8, 20] by [−3, 5]
(x_4, y_4)



[−8, 20] by [−3, 5]
(x_5, y_5)



[−2, 5] by [−10, 20]
(x_6, y_6)

For (x_4, y_4) and (x_5, y_5), the particle is moving to the right when the x-coordinate of the graph (velocity) is positive, moving to the left when the x-coordinate of the graph (velocity) is negative, and is stopped when the x-coordinate of the graph (velocity) is 0. For (x_6, y_6), the particle is moving to the right when the y-coordinate of the graph (velocity) is positive, moving to the left when the y-coordinate of the graph (velocity) is negative, and is stopped when the y-coordinate of the graph (velocity) is 0.

Exploration 3 Seeing Motion on a Graphing Calculator

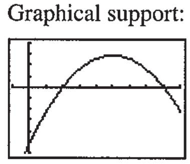
1. Let tMin = 0 and tMax = 10.

2. Since the rock achieves a maximum height of 400 feet, set yMax to be slightly greater than 400, for example yMax = 420.

4. The grapher proceeds with constant increments of t (time), so pixels appear on the screen at regular time intervals. When the rock is moving more slowly, the pixels appear closer together. When the rock is moving faster, the pixels appear farther apart. We observe faster motion when the pixels are farther apart.

Quick Review 3.4

1. The coefficient of x^2 is negative, so the parabola opens downward.
Graphical support:



[−1, 9] by [−300, 200]

2. The y-intercept is $f(0) = -256$.
See the solution to Exercise 1 for graphical support.

3. The x-intercepts occur when $f(x) = 0$.

$$-16x^2 + 160x - 256 = 0$$
$$-16(x^2 - 10x + 16) = 0$$
$$-16(x-2)(x-8) = 0$$
$$x = 2 \text{ or } x = 8$$

The x-intercepts are 2 and 8. See the solution to Exercise 1 for graphical support.

4. Since $f(x) = -16(x^2 - 10x + 16)$
$= -16(x^2 - 10x + 25 - 9) = -16(x-5)^2 + 144$, the range is $(-\infty, 144]$.
See the solution to Exercise 1 for graphical support.

5. Since $f(x) = -16(x^2 - 10x + 16)$
$= -16(x^2 - 10x + 25 - 9) = -16(x-5)^2 + 144$, the vertex is at $(5, 144)$. See the solution to Exercise 1 for graphical support.

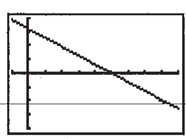
6.
$$f(x) = 80$$
$$-16x^2 + 160x - 256 = 80$$
$$-16x^2 + 160x - 336 = 0$$
$$-16(x^2 - 10x + 21) = 0$$
$$-16(x-3)(x-7) = 0$$
$$x = 3 \text{ or } x = 7$$

$f(x) = 80$ at $x = 3$ and at $x = 7$.
See the solution to Exercise 1 for graphical support.

7.
$$\frac{dy}{dx} = 100$$
$$-32x + 160 = 100$$
$$60 = 32x$$
$$x = \frac{15}{8}$$

$\frac{dy}{dx} = 100$ at $x = \frac{15}{8}$

Graphical support: the graph of
$\text{NDER}(-16x^2 + 160x - 256, x, x)$ is shown.



$[-1, 9]$ by $[-200, 200]$

8.
$$\frac{dy}{dx} > 0$$
$$-32x + 160 > 0$$
$$-32x > -160$$
$$x < 5$$

$\frac{dy}{dx} > 0$ when $x < 5$.
See the solution to Exercise 7 for graphical support.

9. Note that $f'(x) = -32x + 160$.

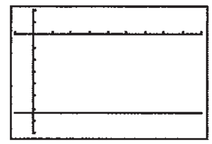
$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = f'(3) = -32(3) + 160 = 64$$

For graphical support, use the graph shown in the solution to Exercise 7 and observe that $\text{NDER}(f(x), 3) \approx 64$.

10. $f'(x) = -32x + 160 \qquad f''(x) = -32$
At $x = 7$ (and, in fact, at any other value of x),

$$\frac{d^2y}{dx^2} = -32.$$

Graphical support: the graph of
$\text{NDER}(\text{NDER}(-16x^2 + 160x - 256,\ x,\ x), x, x)$ is shown.



$[-1, 9]$ by $[-40, 10]$

Section 3.4 Exercises

1. (a) $V(s) = s^3$

(b) $\frac{dV}{ds} = 3s^2$

(c) $\left.\frac{dV}{ds}\right|_{s=1} = 3(1)^2 = 3 \qquad \left.\frac{dV}{ds}\right|_{s=5} = 3(5)^2 = 75$

(d) $\frac{\text{in}^3}{\text{in}}$.

2. (a) $C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$

$$A = \pi r^2 = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$
$$A(C) = \frac{C^2}{4\pi}$$

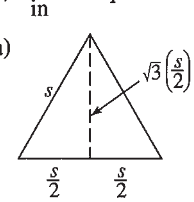
(b) $\frac{dA}{dC} = \frac{2C}{4\pi} = \frac{C}{2\pi}$

(c) $\left.\frac{dA}{dC}\right|_{C=\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$

$\left.\frac{dA}{dC}\right|_{C=6\pi} = \frac{6\pi}{2\pi} = 3$

(d) $\frac{\text{in}^2}{\text{in}}$ or square inches per inch

3. (a)



$$A = \frac{1}{2}bh = \frac{1}{2}s\,\sqrt{3}\left(\frac{s}{2}\right)$$
$$A(s) = \frac{\sqrt{3}}{4}s^2$$

(b) $\frac{dA}{ds} = \frac{\sqrt{3}}{2}s$

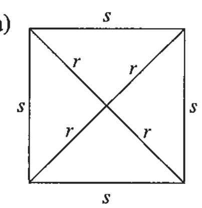
3. Continued

(c) $\left.\dfrac{dA}{ds}\right|_{s=2} = \dfrac{\sqrt{3}}{2}(2) = \sqrt{3}$

$\left.\dfrac{dA}{ds}\right|_{s=10} = \dfrac{\sqrt{3}}{2}(10) = 5\sqrt{3}$

(d) $\dfrac{\text{in}^2}{\text{in}}$ or square inches per inch

4. (a)



Use Pythagorean Theorem on lower right triangle:

$$s^2 + s^2 = (2r)^2$$
$$2s^2 = 4r^2$$
$$s^2 = 2r^2$$
$$A = s^2 = 2r^2$$
$$A(r) = 2r^2$$

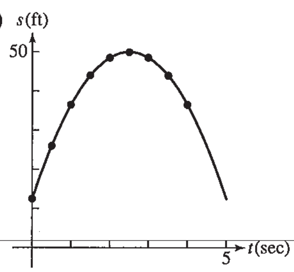
(b) $\dfrac{dA}{dr} = 4r$

(c) $\left.\dfrac{dA}{dr}\right|_{r=1} = 4(1) = 4$

$\left.\dfrac{dA}{dr}\right|_{r=8} = 4(8) = 32$

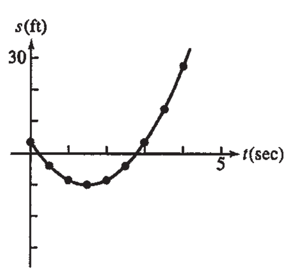
(d) $\dfrac{\text{in}^2}{\text{in}}$ or square inches per inch

5. (a)



(b) $s'(1) = 18$, $s'(2.5) = 0$, $s'(3.5) = -12$

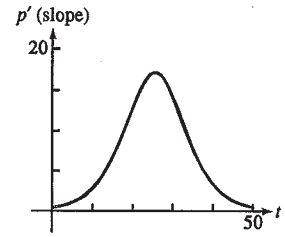
6. (a)



(b) $s'(1) = -6$, $s'(2.5) = 12$, $s'(3.5) = 24$

7. (a) We estimate the slopes at several points as follows, then connect the points to create a smooth curve.

t (days)	0	10	20	30	40	50
Slope (flies/ day)	0.5	3.0	13.0	14.0	3.5	0.5



Horizontal axis: Days
Vertical axis: Flies per day

(b) Fastest: Around the 25th day
Slowest: Day 50 or day 0

8. $Q(t) = 200(30-t)^2 = 200(900 - 60t + t^2)$
$= 180{,}000 - 12{,}000t + 200t^2$
$Q'(t) = -12{,}000 + 400t$

The rate of change of the amount of water in the tank after 10 minutes is $Q'(10) = -8000$ gallons per minute.

Note that $Q'(10) < 0$, so the rate at which the water is running *out* is positive. The water is running out at the rate of 8000 gallons per minute.

The average rate for the first 10 minutes is

$$\frac{Q(10)-Q(0)}{10-0} = \frac{80{,}000-180{,}000}{10} = -10{,}000 \text{ gal/min.}$$

The water is flowing out at an average rate of 10,000 gallons per minute over the first 10 min.

9. (a) The particle moves forward when $v > 0$, for $0 \le t < 1$ and for $5 < t < 7$.
The particle moves backward when $v < 0$, for $1 < t < 5$.
The particle speeds up when v is negative and decreasing, for $1 < t < 2$, and when v is positive and increasing, for $5 < t < 6$.
The particle slows down when v is positive and decreasing, for $0 \le t < 1$ and for $6 < t < 7$, and when v is negative and increasing, for $3 < t < 5$.

(b) Note that the acceleration $a = \dfrac{dv}{dt}$ is undefined at $t = 2$, $t = 3$, and $t = 6$.
The acceleration is positive when v is increasing, for $3 < t < 6$.
The acceleration is negative when v is decreasing, for $0 \le t < 2$ and for $6 < t < 7$.
The acceleration is zero when v is constant, for $2 < t < 3$ and for $7 < t \le 9$.

(c) The particle moves at its greatest speed when $|v|$ is maximized, at $t = 0$ and for $2 < t < 3$.

(d) The particle stands still for more than an instant when v stays at zero, for $7 < t \le 9$.

10. **(a)** The particle is moving left when the graph of s has negative slope, for $2 < t < 3$ and for $5 < t \le 6$.
The particle is moving right when the graph of s has positive slope, for $0 \le t < 1$.
The particle is standing still when the graph of s is horizontal, for $1 < t < 2$ and for $3 < t < 5$.

(b) For $0 \le t < 1$: $v = \dfrac{2-0}{1-0} = 2$ cm/sec

Speed $= |v| = 2$ cm/sec

For $1 < t < 2$: $v = \dfrac{2-2}{2-1} = 0$ cm/sec

Speed $= |v| = 0$ cm/sec

For $2 < t < 3$: $v = \dfrac{-2-2}{3-2} = -4$ cm/sec

Speed $= |v| = 4$ cm/sec

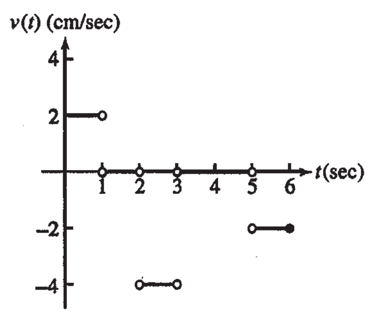
For $3 < t < 5$: $v = \dfrac{-2-(-2)}{5-3} = 0$ cm/sec

Speed $= |v| = 0$ cm/sec

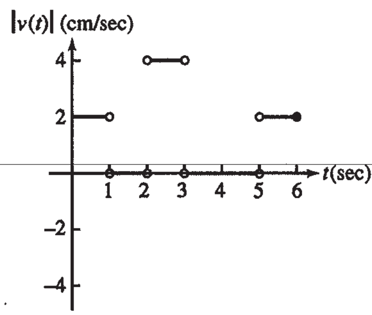
For $5 < t \le 6$: $v = \dfrac{-4-(-2)}{6-5} = -2$ cm/sec

Speed $= |v| = 2$ cm/sec

Velocity graph:



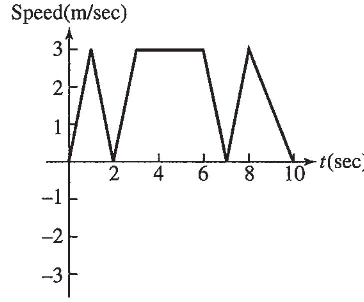
Speed graph:



11. **(a)** The body reverses direction when v changes sign, at $t = 2$ and at $t = 7$.

(b) The body is moving at a constant speed, $|v| = 3$ m/sec, between $t = 3$ and $t = 6$.

(c) The speed graph is obtained by reflecting the negative portion of the velocity graph, $2 < t < 7$, about the x-axis.



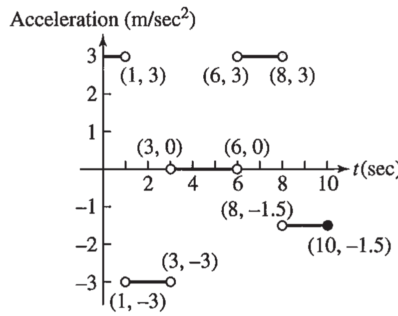
(d) For $0 \le t < 1$: $a = \dfrac{3-0}{1-0} = 3$ m/sec^2

For $1 < t < 3$: $a = \dfrac{-3-3}{3-1} = -3$ m/sec^2

For $3 < t < 6$: $a = \dfrac{-3-(-3)}{6-3} = 0$ m/sec^2

For $6 < t < 8$: $a = \dfrac{3-(-3)}{8-6} = 3$ m/sec^2

For $8 < t \le 10$: $a = \dfrac{0-3}{10-8} = -1.5$ m/sec^2



12. **(a)** It takes 135 seconds.

(b) Average speed $= \dfrac{\Delta F}{\Delta t} = \dfrac{5-0}{73-0} = \dfrac{5}{73}$
≈ 0.068 furlongs/sec.

(c) Using a symmetric difference quotient, the horse's speed is approximately
$\dfrac{\Delta F}{\Delta t} = \dfrac{4-2}{59-33} = \dfrac{2}{26} = \dfrac{1}{13} \approx 0.077$ furlongs/sec.

(d) The horse is running the fastest during the last furlong (between 9th and 10th furlong markers). This furlong takes only 11 seconds to run, which is the least amount of time for a furlong.

(e) The horse accelerates the fastest during the first furlong (between markers 0 and 1).

13. **(a)** Velocity: $v(t) = \dfrac{ds}{dt} = \dfrac{d}{dt}(24t - 0.8t^2) = 24 - 1.6t$ m/sec

Acceleration: $a(t) = \dfrac{dv}{dt} = \dfrac{d}{dt}(24 - 1.6t) = -1.6$ m/sec^2

(b) The rock reaches its highest point when $v(t) = 24 - 1.6t = 0$, at $t = 15$. It took 15 seconds.

(c) The maximum height was $s(15) = 180$ meters.

13. Continued

(d) $s(t) = \frac{1}{2}(180)$

$$24t - 0.8t^2 = 90$$
$$0 = 0.8t^2 - 24t + 90$$
$$t = \frac{24 \pm \sqrt{(-24)^2 - 4(0.8)(90)}}{2(0.8)}$$
$$\approx 4.393,\ 25.607$$

It took about 4.393 seconds to reach half its maximum height.

(e) $s(t) = 0$

$$24t - 0.8t^2 = 0$$
$$0.8t(30 - t) = 0$$
$$t = 0 \text{ or } t = 30$$

The rock was aloft from $t = 0$ to $t = 30$, so it was aloft for 30 seconds.

14. On Mars:

Velocity $= \frac{ds}{dt} = \frac{d}{dt}(1.86t^2) = 3.72t$

Solving $3.72t = 16.6$, the downward velocity reaches 16.6 m/sec after about 4.462 sec.

On Jupiter:

Velocity $= \frac{ds}{dt} = \frac{d}{dt}(11.44t^2) = 22.88t$

Solving $22.88t = 16.6$, the downward velocity reaches 16.6 m/sec after about 0.726 sec.

15. The rock reaches its maximum height when the velocity $s'(t) = 24 - 9.8t = 0$, at $t \approx 2.449$. Its maximum height is about $s(2.449) \approx 29.388$ meters.

16. Moon:

$$s(t) = 0$$
$$832t - 2.6t^2 = 0$$
$$2.6t(320 - t) = 0$$
$$t = 0 \text{ or } t = 320$$

It takes 320 seconds to return.

Earth:

$$s(t) = 0$$
$$832t - 16t^2 = 0$$
$$16t(52 - t) = 0$$
$$t = 0 \text{ or } t = 52$$

It takes 52 seconds to return.

17. The following is one way to simulate the problem situation.

For the moon:

$x_1(t) = 3(t < 160) + 3.1(t \ge 160)$

$y_1(t) = 832t - 2.6t^2$

t-values: 0 to 320

window: [0, 6] by [−10,000, 70,000]

For the earth:

$x_1(t) = 3(t < 26) + 3.1(t \ge 26)$

$y_1(t) = 832t - 16t^2$

t-values: 0 to 52

window: [0, 6] by [−1000, 11,000]

18. (a) 190 ft/sec

(b) 2 seconds

(c) After 8 seconds, and its velocity was 0 ft/sec then

(d) After about 11 seconds, and it was falling 90 ft/sec then

(e) About 3 seconds (from the rocket's highest point)

(f) The acceleration was greatest just before the engine stopped. The acceleration was constant from $t = 2$ to $t = 11$, while the rocket was in free fall.

19. (a) Displacement: $= s(5) - s(0) = 12 - 2 = 10$ m

(b) Average velocity $= \frac{10 \text{ m}}{5 \text{ sec}} = 2$ m/sec

(c) Velocity $= s'(t) = 2t - 3$

At $t = 4$, velocity $= s'(4) = 2(4) - 3 = 5$ m/sec

(d) Acceleration $= s''(t) = 2$ m/sec^2

(e) The particle changes direction when

$s'(t) = 2t - 3 = 0$, so $t = \frac{3}{2}$ sec.

(f) Since the acceleration is always positive, the position s is at a minimum when the particle changes direction, at $t = \frac{3}{2}$ sec. Its position at this time is $s\left(\frac{3}{2}\right) = -\frac{1}{4}$ m.

20. (a) $v(t) = \frac{ds}{dt} = \frac{d}{dt}(-t^3 + 7t^2 - 14t + 8)$

$v(t) = -3t^2 + 14t - 14$

(b) $a(t) = \frac{dv}{dt} = \frac{d}{dt}(-3t^2 + 14t - 14)$

$a(t) = -6t + 14$

(c) $v(t) = -3t^2 + 14t - 14 = 0$

$t \approx 1.451,\ 3.215$

(d) The particle starts at the point $s = 8$ when $t = 0$ and moves left until it stops at $s = -0.631$ when $t = 1.451$, then it moves right to the point $s = 2.113$ when $t = 3.215$ where it stops again, and finally continues left from there on.

21. (a) $v(t) = \frac{ds}{dt} = \frac{d}{dt}[(t-2)^2(t-4)]$

$$= (t-2)^2(1) + (t-4) \cdot 2(t-2)$$
$$= (t-2)[(t-2) + 2(t-4)]$$
$$= (t-2)(3t-10)$$

(b) $a(t) = \frac{dv}{dt} = \frac{d}{dt}[(t-2)(3t-10)]$

$a(t) = 6t - 16$

21. Continued

(c) $v(t) = (t-2)(3t-10) = 0$

$t = 2, \frac{10}{3}$

(d) The particle starts at the point $s = -16$ when $t = 0$ and move right until it stops at $s = 0$ when $t = 2$, then it moves left to the point $s = -1.185$ when $t = \frac{10}{3}$ where it stops again, and finally continues right from there on.

22. (a) $v(t) = \frac{ds}{dt} = \frac{d}{dt}(t^3 - 6t^2 + 8t + 2)$

$v(t) = 3t^2 - 12t + 8$

(b) $a(t) = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 12t + 8)$

$a(t) = 6t - 12$

(c) $v(t) = 3t^2 - 12t + 8 = 0 \qquad t \approx 0.845,\ 3.155$

(d) The particle starts at the point $s = 2$ when $t = 0$ and moves right until it stops at $s = 5.079$ when $t = 0.845$, then it moves left to the point $s = -1.079$ when $t = 3.155$ where it stops again, and finally continues right from there on.

23. $v(t) = s'(t) = 3t^2 - 12t + 9$

$a(t) = v'(t) = 6t - 12$

Find when velocity is zero.

$3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-1)(t-3) = 0$

$t = 1$ or $t = 3$

At $t = 1$, the acceleration is $a(1) = -6$ m/sec^2

At $t = 3$, the acceleration is $a(3) = 6$ m/sec^2

24. $a(t) = v'(t) = 6t^2 - 18t + 12$

Find when acceleration is zero.

$6t^2 - 18t + 12 = 0$

$6(t^2 - 3t + 2) = 0$

$6(t-1)(t-2) = 0$

$t = 1$ or $t = 2$

At $t = 1$, the speed is $|v(1)| = |0| = 0$ m/sec.

At $t = 2$, the speed is $|v(2)| = |-1| = 1$ m/sec.

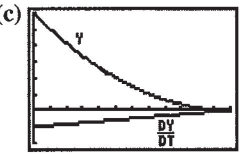
25. (a) $\frac{dy}{dt} = \frac{d}{dt}\left[6\left(1 - \frac{t}{12}\right)^2\right]$

$= \frac{d}{dt}\left[6\left(1 - \frac{t}{6} + \frac{t^2}{144}\right)\right] = \frac{d}{dt}\left(6 - t + \frac{1}{24}t^2\right)$

$= 0 - 1 + \frac{t}{12} = \frac{t}{12} - 1$

(b) The fluid level is falling fastest when $\frac{dy}{dt}$ is the most negative, at $t = 0$, when $\frac{dy}{dt} = -1$. The fluid level is falling slowest at $t = 12$, when $\frac{dy}{dt} = 0$.

(c)

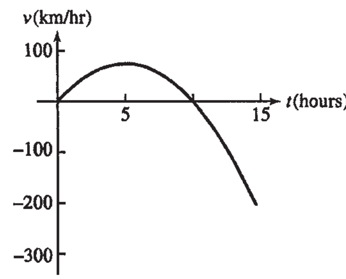


[0, 12] by [−2, 6]

y is decreasing and $\frac{dy}{dt}$ is negative over the entire interval y decreases more rapidly early in the interval, and the magnitude of $\frac{dy}{dt}$ is larger then. $\frac{dy}{dt}$ is 0 at $t = 12$, where the graph of y seems to have a horizontal tangent.

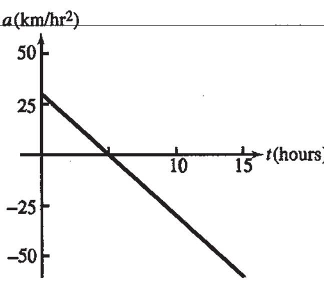
26. (a) To graph the velocity, we estimate the slopes at several points as follows, then connect the points to create a smooth curve.

t (hours)	0	2.5	5	7.5	10	12.5	15
v (km/hour)	0	56	75	56	0	−94	−225



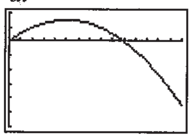
To graph the acceleration, we estimate the slope of the velocity graph at several points as follows, and then connect the points to create a smooth curve.

t (hours)	0	2.5	5	7.5	10	12.5	15
a (km/hour^2)	30	15	0	−15	−30	−45	−60



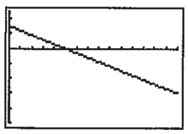
26. Continued

(b) $\frac{ds}{dt} = 30t - 3t^2$



[0, 15] by [–300, 100]

$\frac{d^2s}{dt^2} = 30 - 6t$



[0, 15] by [–100, 50]

The graphs are very similar.

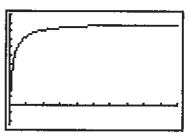
27. (a) Average cost $= \frac{c(100)}{100} = \frac{11{,}000}{100} = \$110$ per machine

(b) $c'(x) = 100 - 0.2x$

Marginal cost $= c'(100) = \$80$ per machine

(c) Actual cost of 101st machine is $c(101) - c(100) = \$79.90$, which is very close to the marginal cost calculated in part (b).

28. (a)



[0, 50] by [–500, 2200]

The values of x which make sense are the whole numbers, $x \geq 0$.

(b) Marginal revenue $= r'(x) = \frac{d}{dx}\left[2000\left(1 - \frac{1}{x+1}\right)\right]$

$$= \frac{d}{dx}\left(2000 - \frac{2000}{x+1}\right)$$

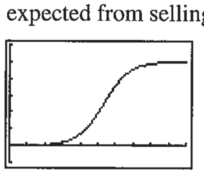
$$= 0 - \frac{(x+1)(0) - (2000)(1)}{(x+1)^2} = \frac{2000}{(x+1)^2}$$

(c) $r'(5) = \frac{2000}{(5+1)^2} = \frac{2000}{36} \approx 55.56$

The increase in revenue is approximately \$55.56.

(d) The limit is 0. This means that as x gets large, one reaches a point where very little extra revenue can be expected from selling more desks.

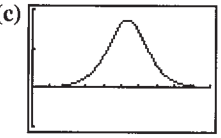
29. (a)



[0, 200] by [–2, 12]

(b) The values of x which make sense are the whole numbers, $x \geq 0$.

(c)



[0, 200] by [–0.1, 0.2]

P is most sensitive to changes in x when $|P'(x)|$ is largest. It is relatively sensitive to changes in x between approximately $x = 60$ and $x = 160$.

(d) The marginal profit, $P'(x)$, is greatest at $x = 106.44$. Since x must be an integer, $P(106) \approx 4.924$ thousand dollars or \$4924.

(e) $P'(50) \approx 0.013$, or \$13 per package sold

$P'(100) \approx 0.165$, or \$165 per package sold

$P'(125) \approx 0.118$, or \$118 per package sold

$P'(150) \approx 0.031$, or \$31 per package sold

$P'(175) \approx 0.006$, or \$6 per package sold

$P'(300) \approx 10^{-6}$, or \$0.001 per package sold

(f) The limit is 10. The maximum possible profit is \$10,000 monthly.

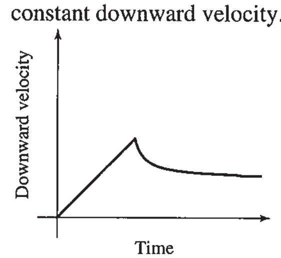
(g) Yes. In order to sell more and more packages, the company might need to lower the price to a point where they won't make any additional profit.

30. Since the particle moves along the line $y = 2$, it will be at the point (5, 2) when $x(t) = 4t^3 - 16t^2 + 15t = 5$. Use a grapher to see that this occurs when $t = 2.83$.

31. Graph C is position, graph A is velocity, and graph B is acceleration.
A is the derivative of C because it is positive, negative, and zero where C is increasing, decreasing, and has horizontal tangents, respectively. The relationship between B and A is similar.

32. Graph C is position, graph B is velocity, and graph A is acceleration.
B is the derivative of C because it is negative and zero where C is decreasing and has horizontal tangents, respectively.
A is the derivative of B because it is positive, negative, and zero where B is increasing, decreasing, and has horizontal tangents, respectively.

33. Note that "downward velocity" is positive when McCarthy is falling downward. His downward velocity increases steadily until the parachute opens, and then decreases to a constant downward velocity. One possible sketch:



34. **(a)** $\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$

When $r = 2$, $\frac{dv}{dr} = 4\pi(2)^2 = 16\pi$ cubic feet of volume per foot of radius.

(b) The increase in the volume is

$\frac{4}{3}\pi(2.2)^3 - \frac{4}{3}\pi(2)^3 \approx 11.092$ cubic feet.

35. Let v_0 be the exit velocity of a particle of lava. Then $s(t) = v_0 t - 16t^2$ feet, so the velocity is

$\frac{ds}{dt} = v_0 - 32t$. Solving $\frac{ds}{dt} = 0$ gives $t = \frac{v_0}{32}$. Then the maximum height, in feet, is

$s\left(\frac{v_0}{32}\right) = v_0\left(\frac{v_0}{32}\right) - 16\left(\frac{v_0}{32}\right)^2 = \frac{v_0^{\,2}}{64}$. Solving

$\frac{v_0^{\,2}}{64} = 1900$ gives $v_0 \approx \pm 348.712$. The exit velocity was

about 348.712 ft/sec. Multiplying by $\frac{3600 \text{ sec}}{1 \text{ h}} \bullet \frac{1 \text{ mi}}{5280 \text{ ft}}$, we find that this is equivalent to about 237.758 mi/h.

36. By estimating the slope of the velocity graph at that point.

37. The motion can be simulated in parametric mode using $x_1(t) = 2t^3 - 13t^2 + 22t - 5$ and $y_1(t) = 2$ in $[-6, 8]$ by $[-3, 5]$.

(a) It begins at the point $(-5, 2)$ moving in the positive direction. After a little more than one second, it has moved a bit past $(6, 2)$ and it turns back in the negative direction for approximately 2 seconds. At the end of that time, it is near $(-2, 2)$ and it turns back again in the positive direction. After that, it continues moving in the positive direction indefinitely, speeding up as it goes.

(b) The particle speeds up when its *speed* is increasing, which occurs during the approximate intervals $1.153 \le t \le 2.167$ and $t \ge 3.180$. It slows down during the approximate intervals $0 \le t \le 1.153$ and $2.167 \le t \le 3.180$. One way to determine the endpoints of these intervals is to use a grapher to find the minimums and maximums for the speed,

$\left|\frac{dx}{dt}\right| = \left|6t^2 - 26t + 22\right|$ using function mode in the window $[0, 5]$ by $[0, 10]$.

(c) The particle changes direction at $t \approx 1.153$ sec and at $t \approx 3.180$ sec.

(d) The particle is at rest "instantaneously" at $t \approx 1.153$ sec and at $t \approx 3.180$ sec.

(e) The velocity starts out positive but decreasing, it becomes negative, then starts to increase, and becomes positive again and continues to increase.
The speed is decreasing, reaches 0 at $t \approx 1.15$ sec, then increases until $t \approx 2.17$ sec, decreases until $t \approx 3.18$ sec when it is 0 again, and then increases after that.

(f) The particle is at (5, 2) when $2t^3 - 13t^2 + 22t - 5 = 5$ at $t \approx 0.745$ sec, $t \approx 1.626$ sec, and at $t \approx 4.129$ sec.

38. **(a)** Solving $160 = 490t^2$ gives $t = \pm\frac{4}{7}$. It took $\frac{4}{7}$ of a second.

The average velocity was $\frac{160 \text{ cm}}{\left(\frac{4}{7}\right) \text{ sec}} = 280$ cm/sec.

(b) $V = \frac{ds}{dt} = 980t$

$a = \frac{dV}{dt} = 980$

At $s = 160$ cm, $t = \frac{4}{7}$ sec (from part (*a*)) and

$V = 980\left(\frac{4}{7}\right) = 560$ cm/sec

$a = 980$ cm/sec^2

(c) Once the balls begin falling, each flash will produce a different image. There are 16 images of the balls falling, so $\frac{16 \text{ flashes}}{4/7 \text{ seconds}} = 28$ flashes per second.

39. Since profit = revenue − cost, the Sum and Difference Rule gives $\frac{d}{dx}(\text{profit}) = \frac{d}{dx}(\text{revenue}) - \frac{d}{dx}(\text{cost})$, where x is the number of units produced. This means that marginal profit = marginal revenue − marginal cost.

40. False. It is the absolute value of the velocity.

41. True. The acceleration is the first derivative of the velocity which, in turn, is the second derivative of the position function.

42. C. $f'(x) = 2x + \frac{2}{x^2}$

$f'(-1) = 2(-1) + \frac{2}{(-1)^2} = 0$

43. D. $V(x) = x^3$

$\frac{dv}{dx} = 3x^2$

44. E. $\frac{ds}{dt} = \frac{d}{dt}(2 + 7t - t^2)$

$v(t) = 7 - 2t < 0$

$7 < 2t$

$\frac{7}{2} < t$

$4 > \frac{7}{2}$

45. C. $v(t) = 7 - 2t = 0$

$7 = 2t$

$t = \frac{7}{2}$

46. The growth rate is given by

$b'(t) = 10^4 - 2 \cdot 10^3(t) = 10{,}000 - 2000t.$

At $t = 0$: $b'(0) = 10{,}000$ bacteria/hour

At $t = 5$: $b'(5) = 0$ bacteria/hour

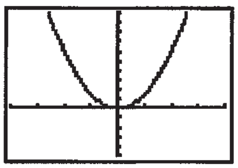
At $t = 10$: $b'(10) = -10{,}000$ bacteria/hour

47. (a) $g'(x) = \frac{d}{dx}(x^3) = 3x^2$

$h'(x) = \frac{d}{dx}(x^3 - 2) = 3x^2$

$t'(x) = \frac{d}{dx}(x^3 + 3) = 3x^2$

(b) The graphs of NDER $g(x)$, NDER $h(x)$, and NDER $t(x)$ are all the same, as shown.



[−4, 4] by [−10, 20]

(c) $f(x)$ must be of the form $f(x) = x^3 + c$, where c is a constant.

(d) Yes. $f(x) = x^3$

(e) Yes. $f(x) = x^3 + 3$

48. For $t > 0$, the speed of the aircraft in meters per second after t seconds is $v(t) = \frac{20}{9}t$. Multiplying by $\frac{3600 \text{ sec}}{1 \text{ h}} \cdot \frac{1 \text{ km}}{1000 \text{ m}}$, we find that this is equivalent to $8t$ kilometers per hour. Solving $8t = 200$ gives $t = 25$ seconds. The aircraft takes 25 seconds to become airborne, and the distance it travels during this time is $D(25) \approx 694.444$ meters.

49. (a) Assume that f is even. Then,

$$f'(-x) = \lim_{h\to 0}\frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0}\frac{f(x-h) - f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0}\frac{f(x+k) - f(x)}{-k} = -\lim_{k\to 0}\frac{f(x+k) - f(x)}{k} = -f'(x)$$

So, f' is an odd function.

(b) Assume that f is odd. Then,

$$f'(-x) = \lim_{h\to 0}\frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0}\frac{-f(x-h) + f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0}\frac{-f(x+k) + f(x)}{-k} = \lim_{k\to 0}\frac{f(x+k) - f(x)}{k} = f'(x)$$

So, f' is an even function.

50. $$\frac{d}{dx}(fgh) = \frac{d}{dx}[f(gh)] = f \cdot \frac{d}{dx}(gh) + gh \cdot \frac{d}{dx}(f)$$

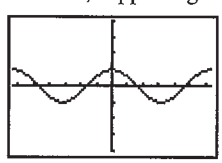
$$= f\left(g \cdot \frac{dh}{dx} + h \cdot \frac{dg}{dx}\right) + gh \cdot \frac{df}{dx}$$

$$= \left(\frac{df}{dx}\right)gh + f\left(\frac{dg}{dx}\right)h + fg\left(\frac{dh}{dx}\right)$$

Section 3.5 Derivatives of Trigonometric Functions (pp. 141–147)

Exploration 1 Making a Conjecture with NDER

1. When the graph of $\sin x$ is increasing, the graph of NDER ($\sin x$) is positive (above the x-axis).
2. When the graph of $\sin x$ is decreasing, the graph of NDER ($\sin x$) is negative (below the x-axis).
3. When the graph of $\sin x$ stops increasing and starts decreasing, the graph of NDER ($\sin x$) crosses the x-axis from above to below.
4. The slope of the graph of $\sin x$ matches the value of NDER ($\sin x$) at these points.
5. We conjecture that NDER ($\sin x$) = $\cos x$. The graphs coincide, supporting our conjecture.



[-2π, 2π] by [−4, 4]

6. When the graph of $\cos x$ is increasing, the graph of NDER ($\cos x$) is positive (above the x-axis).
When the graph of $\cos x$ is decreasing, the graph of NDER ($\cos x$) is negative (below the x-axis).
When the graph of $\cos x$ stops increasing and starts decreasing, the graph of NDER ($\cos x$) crosses the x-axis from above to below.
The slope of the graph of $\cos x$ matches the value of NDER ($\cos x$) at these points.