39. Continued

(b) Since the function needs to be continuous, we may assume that $a + b = 2$ and $f(1) = 2$.

$$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{(3-(1+h))-2}{h} = \lim_{h\to 0^-}(-1) = -1$$

$$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{a(1+h)^2+b(1+h)-2}{h}$$
$$= \lim_{h\to 0^+} \frac{a+2ah+ah^2+b+bh-2}{h}$$
$$= \lim_{h\to 0^+} \frac{2ah+ah^2+bh+(a+b-2)}{h}$$
$$= \lim_{h\to 0^+}(2a+ah+b)$$
$$= 2a+b$$

Therefore, $2a + b = -1$. Substituting $2 - a$ for b gives $2a + (2 - a) = -1$, so $a = -3$.
Then $b = 2 - a = 2 - (-3) = 5$. The values are $a = -3$ and $b = 5$.

40. True. See Theorem 1.

41. False. The function $f(x) = |x|$ is continuous at $x = 0$ but is not differentiable at $x = 0$.

42. B.

43. A. $\text{NDER}(f, x, a) = \dfrac{f(a+h)-f(a-h)}{2h}$

$$= \frac{\sqrt[3]{1.001-1}-\sqrt[3]{0.999-1}}{0.002} = 100$$

The symmetric difference quotient gets larger as h gets smaller, so $f'(1)$ is undefined.

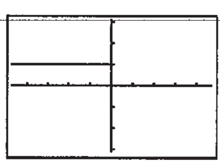
44. B. $\displaystyle\lim_{h\to 0^-} \frac{2(0+h)+1-(2(0)+1)}{h}$

$$= \lim_{h\to 0^-} \frac{2h}{h} = 2$$

45. C. $\displaystyle\lim_{h\to 0^+} \frac{(0+h)^2+1-(0^2+1)}{h}$

$$= \lim_{h\to 0^+} \frac{h^2}{h} = 0$$

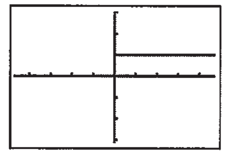
46. (a)



[−4.7, 4.7] by [−3.1, 3.1]

(b) You can use Trace to help see that the value of Y1 is 1 for every $x < 0$ and is 0 for every $x \ge 0$. It appears to be the graph of $f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \ge 0. \end{cases}$

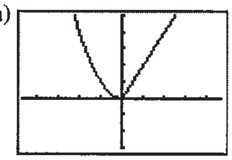
(c)



[−4.7, 4.7] by [−3.1, 3.1]

(d) You can use Trace to help see that the value of Y1 is 0 for every $x < 0$ and is 1 for every $x \ge 0$. It appears to be the graph of $f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \ge 0. \end{cases}$

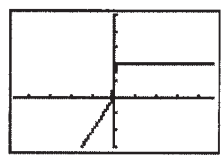
47. (a)



[−4.7, 4.7] by [−3, 5]

(b) See exercise 46.

(c)



[−4.7, 4.7] by [−3, 5]

(d) NDER (Y1, x, −0.1) = −0.1, NDER(Y1, x, 0) = 0.9995, NDER (Y1, x, 0.1) = 2.

48. (a) Note that $-|x| \le x\sin(1/x) \le |x|$ for all x except 0, so $\displaystyle\lim_{x\to 0}\left(x\sin\frac{1}{x}\right) = 0$ by the Sandwich Theorem.
Therefore, f is continuous at $x = 0$.

(b) $\dfrac{f(0+h)-f(0)}{h} = \dfrac{h\sin\frac{1}{h}-0}{h} = \sin\dfrac{1}{h}$

(c) The limit does not exist because $\sin\dfrac{1}{h}$ oscillates between −1 and 1 an infinite number of times arbitrarily close to $h = 0$ (that is, for h in any open interval containing 0).

(d) No, because the one-sided limits (as in part (c)) do not exist.

(e) $\dfrac{g(0+h)-g(0)}{h} = \dfrac{h^2\sin\left(\frac{1}{h}\right)-0}{h} = h\sin\dfrac{1}{h}$

As noted in part (a), the limit of this as h approaches zero is 0, so $g'(0) = 0$.

Section 3.3 Rules for Differentiation (pp. 116–126)

Quick Review 3.3

1. $(x^2-2)(x^{-1}+1) = x^2x^{-1} + x^2 \cdot 1 - 2x^{-1} - 2 \cdot 1$
$= x + x^2 - 2x^{-1} - 2$

2. $\left(\frac{x}{x^2+1}\right)^{-1} = \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + x^{-1}$

3. $3x^2 - \frac{2}{x} + \frac{5}{x^2} = 3x^2 - 2x^{-1} + 5x^{-2}$

4. $\frac{3x^4 - 2x^3 + 4}{2x^2} = \frac{3x^4}{2x^2} - \frac{2x^3}{2x^2} + \frac{4}{2x^2}$

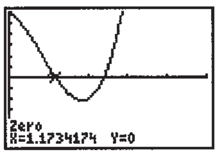
$= \frac{3}{2}x^2 - x + 2x^{-2}$

5. $(x^{-1}+2)(x^{-2}+1) = x^{-1}x^{-2} + x^{-1} \bullet 1 + 2x^{-2} + 2 \bullet 1$

$= x^{-3} + x^{-1} + 2x^{-2} + 2$

6. $\frac{x^{-1}+x^{-2}}{x^{-3}} = x^3(x^{-1}+x^{-2}) = x^2 + x$

7.



[0, 5] by [−6, 6]

At $x \approx 1.173$, $500x^6 \approx 1305$.
At $x \approx 2.394$, $500x^6 \approx 94{,}212$.
After rounding, we have:
At $x \approx 1$, $500x^6 \approx 1305$.
At $x \approx 2$, $500x^6 \approx 94{,}212$.

8. (a) $f(10) = 7$

(b) $f(0) = 7$

(c) $f(x+h) = 7$

(d) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{7-7}{x-a} = \lim_{x\to a} 0 = 0$

9. These are all constant functions, so the graph of each function is a horizontal line and the derivative of each function is 0.

10. (a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h}{\pi} - \frac{x}{\pi}}{h}$

$= \lim_{h\to 0} \frac{x+h-x}{\pi h} = \lim_{h\to 0} \frac{1}{\pi} = \frac{1}{\pi}$

(b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{\pi}{x+h} - \frac{\pi}{x}}{h}$

$= \lim_{h\to 0} \frac{\pi x - \pi(x+h)}{hx(x+h)} = \lim_{h\to 0} \frac{-\pi h}{hx(x+h)}$

$= \lim_{h\to 0} -\frac{\pi}{x(x+h)} = -\frac{\pi}{x^2} = -\pi x^{-2}$

Section 3.3 Exercises

1. $\frac{dy}{dx} = \frac{d}{dx}(-x^2) + \frac{d}{dx}(3) = -2x + 0 = -2x$

2. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}(x) = x^2 - 1$

3. $\frac{dy}{dx} = \frac{d}{dx}(2x) + \frac{d}{dx}(1) = 2 + 0 = 2$

4. $\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1) = 2x + 1 + 0 = 2x + 1$

5. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3\right) + \frac{d}{dx}\left(\frac{1}{2}x^2\right) + \frac{d}{dx}(x)$

$= x^2 + x + 1$

6. $\frac{dy}{dx} = \frac{d}{dx}(1) - \frac{d}{dx}(x) + \frac{d}{dx}(x^2) - \frac{d}{dx}(x^3)$

$= 0 - 1 + 2x - 3x^2 = -1 + 2x - 3x^2$

7. $\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x^2 + x + 1)$

$= 3x^2 - 4x + 1 = 0$

$x = \frac{1}{3}, 1$

8. $\frac{dy}{dx} = \frac{d}{dx}(x^3 - 4x^2 + x + 2)$

$= 3x^2 - 8x + 1 = 0$

$x = \frac{4-\sqrt{13}}{3}, \frac{4+\sqrt{13}}{3}$ or

$\approx 0.131, 2.535$

9. $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 4x^2 + 1)$

$= 4x^3 - 8x = 0$

$x = 0, \pm\sqrt{2}$

10. $\frac{dy}{dx} = \frac{d}{dx}(4x^3 - 6x^2 - 1)$

$= 12x^2 - 12x = 0$

$x = 0, 1$

11. $\frac{dy}{dx} = \frac{d}{dx}(5x^3 - 3x^5)$

$= 15x^2 - 15x^4 = 0$

$x = -1, 0, 1$

12. $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 7x^3 + 2x^2 + 15)$

$= 4x^3 - 21x^2 + 4x = 0$

$x = 0, \frac{21-\sqrt{377}}{8} \approx 0.198, \frac{21+\sqrt{377}}{8} \approx 5.052$

13. (a) $\frac{dy}{dx} = \frac{d}{dx}[(x+1)(x^2+1)]$

$= (x+1)\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}(x+1)$

$= (x+1)(2x) + (x^2+1)(1)$

$= 2x^2 + 2x + x^2 + 1$

$= 3x^2 + 2x + 1$

13. Continued

(b) $\frac{dy}{dx}=\frac{d}{dx}[(x+1)(x^2+1)]$

$=\frac{d}{dx}(x^3+x^2+x+1)$

$=3x^2+2x+1$

14. (a) $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+3}{x}\right)$

$=\frac{x\frac{d}{dx}(x^2+3)-(x^2+3)\frac{d}{dx}(x)}{x^2}$

$=\frac{x(2x)-(x^2+3)}{x^2}$

$=\frac{x^2-3}{x^2}$

(b) $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+3}{x}\right)=\frac{d}{dx}(x+3x^{-1})=1-3x^{-2}$

$=1-\frac{3}{x^2}$

This is equivalent to the answer in part (a).

15. $(x^3+x+1)(x^4+x^2+1)$

$\frac{d}{dx}(x^7+2x^5+x^4+2x^3+x^2+x+1)$

$=7x^6+10x^4+4x^3+6x^2+2x+1$

16. $(x^2+1)(x^3+1)$

$\frac{d}{dx}(x^5+x^3+x^2+1)$

$=5x^4+3x^2+2x$

17. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2x+5}{3x-2}\right)=\frac{(3x-2)(2)-(2x+5)(3)}{(3x-2)^2}=-\frac{19}{(3x-2)^2}$

18. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+5x-1}{x^2}\right)=\frac{d}{dx}(1+5x^{-1}-x^{-2})$

$=0-5x^{-2}+2x^{-3}=-\frac{5}{x^2}+\frac{2}{x^3}$

19. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{(x-1)(x^2+x+1)}{x^3}\right)=\frac{d}{dx}\left(\frac{x^3-1}{x^3}\right)$

$=\frac{d}{dx}(1-x^{-3})=0+3x^{-4}=\frac{3}{x^4}$

20. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1-x}{1+x^2}\right)=\frac{(1+x^2)(-1)-(1-x)(2x)}{(1+x^2)^2}$

$=\frac{x^2-2x-1}{(1+x^2)^2}$

21. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2}{1-x^3}\right)=\frac{(1-x^3)(2x)-x^2(-3x^2)}{(1-x^3)^2}=\frac{x^4+2x}{(1-x^3)^2}$

22. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{(x+1)(x+2)}{(x-1)(x-2)}\right)=\frac{d}{dx}\left(\frac{x^2+3x+2}{x^2-3x+2}\right)$

$=\frac{(x^2-3x+2)(2x+3)-(x^2+3x+2)(2x-3)}{(x^2-3x+2)^2}$

$=\frac{(2x^3-3x^2-5x+6)-(2x^3+3x^2-5x-6)}{(x^2-3x+2)^2}$

$=\frac{12-6x^2}{(x^2-3x+2)^2}$

23. (a) At $x=0$, $\frac{d}{dx}(uv)=u(0)v'(0)+v(0)u'(0)$

$=(5)(2)+(-1)(-3)=13$

(b) At $x=0$, $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v(0)u'(0)-u(0)v'(0)}{[v(0)]^2}$

$=\frac{(-1)(-3)-(5)(2)}{(-1)^2}=-7$

(c) At $x=0$, $\frac{d}{dx}\left(\frac{v}{u}\right)=\frac{u(0)v'(0)-v(0)u'(0)}{[u(0)]^2}$

$=\frac{(5)(2)-(-1)(-3)}{(5)^2}=\frac{7}{25}$

(d) At $x=0$, $\frac{d}{dx}(7v-2u)=7v'(0)-2u'(0)$

$=7(2)-2(-3)=20$

24. (a) At $x=2$, $\frac{d}{dx}(uv)=u(2)v'(2)+v(2)u'(2)$

$=(3)(2)+(1)(-4)=2$

(b) At $x=2$, $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v(2)u'(2)-u(2)v'(2)}{[v(2)]^2}$

$=\frac{(1)(-4)-(3)(2)}{(1)^2}=-10$

(c) At $x=2$, $\frac{d}{dx}\left(\frac{v}{u}\right)=\frac{u(2)v'(2)-v(2)u'(2)}{[u(2)]^2}$

$=\frac{(3)(2)-(1)(-4)}{(3)^2}=\frac{10}{9}$

(d) Use the result from part (a) for $\frac{d}{dx}(uv)$.

At $x=2$, $\frac{d}{dx}(3u-2v+2uv)$

$=3u'(2)-2v'(2)+2\frac{d}{dx}(uv)\Big|_{x=2}$

$=3(-4)-2(2)+2(2)$

$=-12$

25. $y'(x)=2x+5$

$y'(3)=2(3)+5=11$

The slope is 11. (iii)

26. The given equation is equivalent to $y=\frac{3}{2}x+6$, so the slope is $\frac{3}{2}$. (iii)

27. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^3+1}{2x}\right)$

$=\frac{(3x^2)2x-2(x^3+1)}{4x^2}=\frac{4x^3-2}{4x^2}$

$y'(1)=\frac{4(1)^3-2}{4(1)^2}=\frac{1}{2}$

$y(1)=\frac{(1)^3+1}{2(1)}=1$

$y=\frac{1}{2}(x-1)+1=\frac{1}{2}x+\frac{1}{2}$

28. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^4+2}{x^2}\right)$

$=\frac{(4x^3)x^2-2x(x^4+2)}{x^4}=\frac{2x^4-4}{x^3}$

$y'(-1)=\frac{2(-1)^4-4}{(-1)^3}=2$

$y(-1)=\frac{(-1)^4+2}{(-1)^2}=3$

$y=2(x+1)+3$

$y=2x+5$

29. $\frac{dy}{dx}=\frac{d}{dx}(4x^{-2}-8x+1)=-8x^{-3}-8$

30. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^{-4}}{4}-\frac{x^{-3}}{3}+\frac{x^{-2}}{2}-x^{-1}+3\right)$

$=-x^{-5}+x^{-4}-x^{-3}+x^{-2}$

31. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)=\frac{d}{dx}\left(\frac{x^{1/2}-1}{x^{1/2}+1}\right)$

$=\frac{(x^{1/2}+1)\frac{1}{2}x^{-1/2}-(x^{1/2}-1)\frac{1}{2}x^{-1/2}}{(x^{1/2}+1)^2}$

$=\frac{\frac{1}{2}x^{-1/2}[(x^{1/2}+1)-(x^{1/2}-1)]}{(x^{1/2}+1)^2}=\frac{\frac{1}{2}x^{-1/2}\bullet 2}{(x^{1/2}+1)^2}$

$=\frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

32. $\frac{dy}{dx}=\frac{d}{dx}\left(2\sqrt{x}-\frac{1}{\sqrt{x}}\right)=\frac{1}{\sqrt{x}}+\frac{1}{2x^{3/2}}$

33. $y\ =x^4+x^3-2x^2+x-5$

$y'\ =4x^3+3x^2-4x+1$

$y''\ =12x^2+6x-4$

$y'''=24x+6$

$y''''=24$

34. $y\ =x^2+x+3$

$y'=2x+1$

$y''=2$

$y'''=0$

$y''''=0$

35. $y=x^{-1}+x^2$

$y'=-x^{-2}+2x$

$y''=2x^{-3}+2$

$y'''=-6x^{-4}$

$y''''=-24x^{-5}$

36. $y\ =\frac{x+1}{x}=\frac{x}{x}+\frac{1}{x}=1+x^{-}$

$y'=-x^{-2}=-\frac{1}{x^2}$

$y''=2x^{-3}=\frac{2}{x^3}$

$y'''=-6x^{-4}=-\frac{6}{x^4}$

$y''''=24x^{-5}=\frac{24}{x^5}$

37. $y'(x)=3x^2-3$

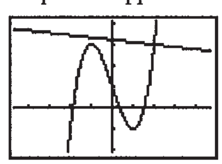
$y'(2)=3(2)^2-3=9$

The tangent line has slope 9, so the perpendicular line has slope $-\frac{1}{9}$ and passes through $(2,3)$.

$y=-\frac{1}{9}(x-2)+3$

$y=-\frac{1}{9}x+\frac{29}{9}$

Graphical support:

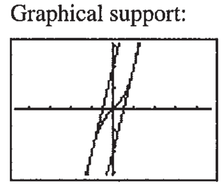


[−4.7, 4.7] by [−2.1, 4.1]

38. $y'(x)=3x^2+1$

The slope is 4 when $3x^2+1=4$, at $x=\pm 1$. The tangent at $x=-1$ has slope 4 and passes through $(-1,-2)$, so its equation is $y=4(x+1)-2$, or $y=4x+2$. The tangent at $x=1$ has slope 4 and passes through $(1,2)$, so its equation is $y=4(x-1)+2$, or $y=4x-2$. The smallest slope occurs when $3x^2+1$ is minimized, so the smallest slope is 1 and occurs at $x=0$.

Graphical support:



[−4.7, 4.7] by [−3.1, 3.1]

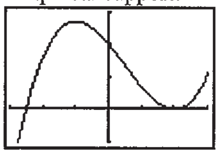
39. $y'(x) = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x+1)(x-2)$

The tangent is parallel to the x-axis when $y' = 0$, at $x = -1$ and at $x = 2$. Since $y(-1) = 27$ and $y(2) = 0$, the two points where this occurs are $(-1, 27)$ and $(2, 0)$.

Graphical support:



$[-3, 3]$ by $[-10, 30]$

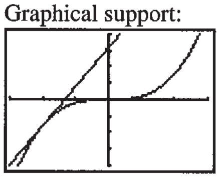
40. $y'(x) = 3x^2$

$y'(-2) = 12$

The tangent line has slope 12 and passes through $(-2, -8)$, so its equation is $y = 12(x+2) - 8$, or $y = 12x + 16$. The

x-intercept is $-\frac{4}{3}$ and the y-intercept is 16.

Graphical support:



$[-3, 3]$ by $[-20, 20]$

41. $y'(x) = \frac{(x^2+1)(4) - 4x(2x)}{(x^2+1)^2} = \frac{-4x^2+4}{(x^2+1)^2}$

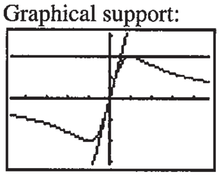
At the origin: $y'(0) = 4$

The tangent is $y = 4x$.

At $(1, 2)$: $y'(1) = 0$

The tangent is $y = 2$.

Graphical support:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

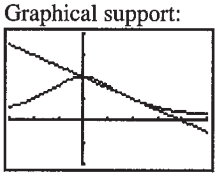
42. $y'(x) = \frac{(4+x^2)(0) - 8(2x)}{(4+x^2)^2} = -\frac{16x}{(4+x^2)^2}$

$y'(2) = -\frac{1}{2}$

The tangent has slope $-\frac{1}{2}$ and passes through $(2, 1)$. Its

equation is $y = -\frac{1}{2}(x-2)+1$, or $y = -\frac{1}{2}x + 2$.

Graphical support:



$[-3, 5]$ by $[-2, 4]$

43. **(a)** Let $f(x) = x$.

$$\frac{d}{dx}(x) = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)-x}{h}$$

$$= \lim_{h\to 0}\frac{h}{h} = \lim_{h\to 0}(1) = 1$$

(b) Note that $u = u(x)$ is a function of x.

$$\frac{d}{dx}(-u) = \lim_{h\to 0}\frac{-u(x+h)-[-u(x)]}{h}$$

$$= \lim_{h\to 0}\left(-\frac{u(x+h)-u(x)}{h}\right)$$

$$= -\lim_{h\to 0}\frac{u(x+h)-u(x)}{h} = -\frac{du}{dx}$$

44. $$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}f(x) + f(x)\cdot\frac{d}{dx}(c)$$

$$= c\cdot\frac{d}{dx}f(x) + 0 = c\cdot\frac{d}{dx}f(x)$$

45. $$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = \frac{f(x)\cdot 0 - 1\cdot\frac{d}{dx}f(x)}{[f(x)]^2} = -\frac{f'(x)}{[f(x)]^2}$$

46. $$\frac{dP}{dV} = \frac{d}{dV}\left(\frac{nRT}{V-nb} - \frac{an^2}{V^2}\right)$$

$$= \frac{(V-nb)\frac{d}{dV}(nRT) - (nRT)\frac{d}{dV}(V-nb)}{(V-nb)^2} - \frac{d}{dV}(an^2V^{-2})$$

$$= \frac{0 - nRT}{(V-nb)^2} + 2an^2V^{-3}$$

$$= -\frac{nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$$

47. $$\frac{ds}{dt} = \frac{d}{dt}(4.9t^2) = 9.8t$$

$$\frac{d^2s}{dt^2} = \frac{d}{dt}(9.8t) = 9.8$$

48. $$\frac{dR}{dM} = \frac{d}{dM}\left[M^2\left(\frac{C}{2} - \frac{M}{3}\right)\right]$$

$$= \frac{d}{dM}\left(\frac{C}{2}M^2 - \frac{1}{3}M^3\right) = CM - M^2$$

49. If the radius of a circle is changed by a very small amount Δr, the change in the area can be thought of as a very thin strip with length given by the circumference, $2\pi r$, and width Δr. Therefore, the change in the area can be approximated by $(2\pi r)(\Delta r)$, which means that the change in the area divided by the change in the radius is approximately $2\pi r$.

50. If the radius of a sphere is changed by a very small amount Δr, the change in the volume can be thought of as a very thin layer with an area given by the surface area, $4\pi r^2$, and a thickness given by Δr. Therefore, the change in the volume can be approximated by $(4\pi r^2)(\Delta r)$, which means that the change in the volume divided by the change in the radius is approximately $4\pi r^2$.

51. Let $t(x)$ be the number of trees and $y(x)$ be the yield per tree x years from now. Then $t(0) = 156$, $y(0) = 12$, $t'(0) = 13$, and $y'(0) = 1.5$. The rate of increase of production is

$$\left.\frac{d}{dx}(ty)\right|_{x=0} = t(0)y'(0) + y(0)t'(0) = (156)(1.5) + (12)(13)$$
$$= 390 \text{ bushels of annual production per year.}$$

52. Let $m(x)$ be the number of members and $c(x)$ be the pavillion cost x years from now. Then $m(0) = 65$, $c(0) = 250$, $m'(0) = 6$, and $c'(0) = 10$. The rate of change of each member's share is

$$\left.\frac{d}{dx}\left(\frac{c}{m}\right)\right|_{x=0} = \frac{m(0)c'(0) - c(0)m'(0)}{[m(0)]^2}$$
$$= \frac{(65)(10) - (250)(6)}{(65)^2} \approx -0.201 \text{ dollars per year. Each}$$

member's share of the cost is decreasing by approximately 20 cents per year.

53. False. π is a constant, so π^3 is also a constant and hence $\frac{d}{dx}(\pi^3) = 0$.

54. True. $f'(x) = -\frac{1}{x^2}$ is never zero, so there are no horizontal tangents.

55. B.
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= (2)(1) + (-1)(3)$$
$$= -1$$

56. D.
$$f(x) = x - \frac{1}{x}$$
$$f'(x) = 1 + \frac{1}{x^2}$$
$$f''(x) = -\frac{2}{x^3}$$

57. E.
$$\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$$
$$= \frac{(x-1) - (x+1)}{(x-1)^2}$$
$$= \frac{-2}{(x-1)^2}$$

58. B.
$$f'(x) = (x^2 - 1) \cdot 2x + (x^2 + 1) \cdot 2x$$
$$= 2x[(x^2 - 1) + (x^2 + 1)]$$
$$= 2x \cdot 2x^2$$
$$= 4x^3$$
$$f'(x) = 0 \text{ only when } x = 0$$

There is one horizontal tangent at $x = 0$.

59. **(a)** It is insignificant in the limiting case and can be treated as zero (and removed from the expression).

(b) It was "rejected" because it is incomparably smaller than the other terms: $v\,du$ and $u\,dv$.

(c) $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$. This is equivalent to the product rule given in the text.

(d) Because dx is "infinitely small," and this could be thought of as dividing by zero.

(e)
$$d\left(\frac{u}{v}\right) = \frac{u + du}{v + dv} - \frac{u}{v}$$
$$= \frac{(u + du)(v) - (u)(v + dv)}{(v + dv)(v)}$$
$$= \frac{uv + vdu - uv - udv}{v^2 + vdv}$$
$$= \frac{vdu - udv}{v^2}$$

Quick Quiz Sections 3.1–3.3

1. D.

2. A. Slope of normal: $m_1 = \frac{1-2}{-1-1} = \frac{1}{2}$

Slope of tangent: $m_2 = -\frac{1}{m_1} = -2$

Therefore $f'(1) = -2$.

3. C.
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{4x-3}{2x+1}\right)$$
$$= \frac{4(2x+1) - 2(4x-3)}{(2x+1)^2}$$
$$= \frac{10}{(2x+1)^2}$$

4. **(a)**
$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 4x^2)$$
$$= 4x^3 - 8x = 4x(x^2 - 2) = 0$$
$$x = 0, \ \pm\sqrt{2}$$

(b)
$$f'(1) = 4(1)(1^2 - 2) = -4$$
$$x = 1 \qquad y = (1)^4 - 4(1)^2$$
$$y = -3$$
$$y = m(x - x_1) + y_1$$
$$y = -4(x - 1) - 3$$
$$y = -4x + 1$$

4. Continued

(c) $m_2 = -\frac{1}{m_1} = \frac{1}{4}$

$$y = \frac{1}{4}(x-1) - 3$$
$$= \frac{1}{4}x - \frac{1}{4} - 3 = \frac{1}{4}x - \frac{13}{4}$$

Section 3.4 Velocity and Other Rates of Change (pp. 127–140)

Exploration 1 Growth Rings on a Tree

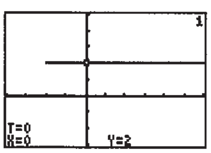
1. Figure 3.22 is a better model, as it shows rings of equal *area* as opposed to rings of equal *width*. It is not likely that a tree could sustain increased growth year after year, although climate conditions do produce some years of greater growth than others.

2. Rings of equal area suggest that the tree adds approximately the same amount of wood to its girth each year. With access to approximately the same raw materials from which to make the wood each year, this is how most trees actually grow.

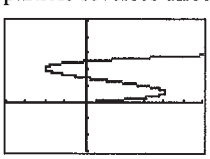
3. Since change in area is constant, so also is $\frac{\text{change in area}}{2\pi}$.
If we denote this latter constant by k, we have
$\frac{k}{\text{change in radius}} = r$, which means that r varies inversely as the change in the radius. In other words, the change in radius must get smaller when r gets bigger, and vice-versa.

Exploration 2 Modeling Horizontal Motion

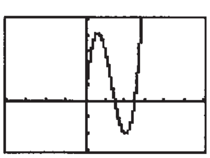
1. The particle reverses direction at about $t = 0.61$ and $t = 2.06$.



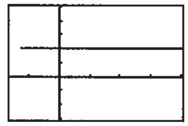
2. When the trace cursor is moving to the right the particle is moving to the right, and when the cursor is moving to the left the particle is moving to the left. Again we find the particle reverses direction at about $t = 0.61$ and $t = 2.06$.



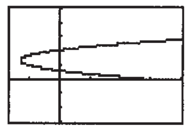
3. When the trace cursor is moving upward the particle is moving to the right, and when the cursor is moving downward the particle is moving to the left. Again we find the same values of t for when the particle reverses direction.



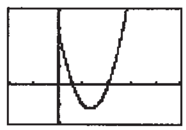
4. We can represent the velocity by graphing the parametric equations
$x_4(t) = x_1'(t) = 12t^2 - 32t + 15,\ y_4(t) = 2$ (part 1),
$x_5(t) = x_1'(t) = 12t^2 - 32t + 15,\ y_5(t) = t$ (part 2),
$x_6(t) = t,\ y_6(t) = x_1'(t) = 12t^2 - 32t + 15$ (part 3)



[−8, 20] by [−3, 5]
(x_4, y_4)



[−8, 20] by [−3, 5]
(x_5, y_5)



[−2, 5] by [−10, 20]
(x_6, y_6)

For (x_4, y_4) and (x_5, y_5), the particle is moving to the right when the x-coordinate of the graph (velocity) is positive, moving to the left when the x-coordinate of the graph (velocity) is negative, and is stopped when the x-coordinate of the graph (velocity) is 0. For (x_6, y_6),the particle is moving to the right when the y-coordinate of the graph (velocity) is positive, moving to the left when the y-coordinate of the graph (velocity) is negative, and is stopped when the y-coordinate of the graph (velocity) is 0.

Exploration 3 Seeing Motion on a Graphing Calculator

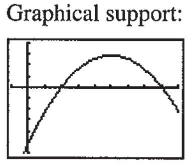
1. Let *t*Min = 0 and *t*Max = 10.

2. Since the rock achieves a maximum height of 400 feet, set *y*Max to be slightly greater than 400, for example *y*Max = 420.

4. The grapher proceeds with constant increments of t (time), so pixels appear on the screen at regular time intervals. When the rock is moving more slowly, the pixels appear closer together. When the rock is moving faster, the pixels appear farther apart. We observe faster motion when the pixels are farther apart.

Quick Review 3.4

1. The coefficient of x^2 is negative, so the parabola opens downward.
Graphical support:



[−1, 9] by [−300, 200]